# Investment Irreversibility, Cash Flow Risk, and Value-Growth Stock Return Effects

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We simulate results from a simple real options model to provide insight into the value-growth stock return anomaly. In our model, firms possess either single ("value" firm) or multiple ("growth" firm) investment opportunities. Our model predicts that growth firms: (1) invest sooner, (2) exhibit greater continuity in capital expenditure over time, (3) have lower book-to-market ratios, and (4) generate lower rates of return than value firms.

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## 1. Introduction

Rates of return for individual stocks are positively related to their book-to-market equity ratios.[1] Stocks with high book-to-market ratios are commonly classified as value stocks, and those with low book-to-market ratios as growth stocks. Rational expectations proponents hypothesize that value stocks have greater distress risk, exhibiting higher average returns as compensation (Fama and French, 1992, 1993, 1995, 1996). In contrast, behavioral finance proponents argue that the value anomaly is a manifestation of systematic mistakes by investors, i.e., over (under) extrapolation of growth (value) firm cash flows (Lakonishok, Shleifer and Vishny, 1994; Haugen, 1995). Despite numerous papers on the subject, the value-growth stock anomaly remains unresolved.[2]

Several rational expectations explanations have been offered for the value-stock premium. For example, Berk, Green, and Naik (1999) investigate relationships between the dynamic turnover of firms' assets and stock return regularities. The authors develop a model in which betas change as assets turn over and as the importance between assets in place and growth options change. In their model, growth firms find valuable projects with low systematic risk. The systematic risk of growth firm cash flows falls, leading to lower expected returns for growth firms relative to value firms.[3] Gomes, Kogan, and Zhang (2003) argue that betas are often

[1] For book-to-market ratio anomalies, see Stattman (1980), Rosenberg, Reid, and Lanstein (1985), Fama and French (1992). Other anomalies include market equity (Banz, 1981), earnings-price ratio (Basu, 1977), and leverage (Bhandari, 1988).
[2] For instance, see Davis, Fama, and French (2000) who test their risk hypothesis against the characteristics hypothesis proposed by Daniel and Titman (1997).
[3] They also suggest that faulty estimates of beta may contribute to the observed effect.

incorrectly estimated. They recommend the use of a conditional CAPM in which conditional betas subsume the firm size and book-to market equity effects.

Carlson, Fisher, and Giammarino (2004) use stochastic product demand and operating leverage to explain the value stock premium. The book-to-market equity ratio, operating leverage, systematic risk, and rates of return rise when demand for the firm’s product decreases. Zhang (2005) uses investment irreversibility and a countercyclical price of risk to explain the value-stock premium. His model predicts that in bad times, value firms suffer from reduced flexibility to scale down unproductive capital. During good times, however, value firms benefit more because they need not incur investment costs, choosing instead to wait until previously unproductive capital becomes productive. Thus, the value premium arises naturally from the higher risk incurred by value firms. Cooper (2006) reaches similar conclusions after modeling a more dynamic adjustment cost process, and providing closed form solutions for risk, book-to-market equity, and the firm’s excess capacity.

We employ a real options model to determine whether option-based simulations produce results consistent with the value-growth anomaly. We identify a firm with a single option to invest as a “value” firm, and we identify a firm with an infinite sequence of options to invest as a “growth” firm. We assume the decision to exercise growth options depends on cash flows from assets in place and cash flows from assets underlying the growth options. Firms will invest only if the present value of expected cash flows from assets underlying the growth options optimally exceeds that of assets-in-place. This assumption differs from Berk, Green, and Naik (1999) and Gomes, Kogan, and Zhang (2003) who assume that investment decisions are unrelated to current cash flows generated by assets in place. Unlike Zhang (2005) and Cooper (2006), our model does not require explicit information about adjustment cost attributes. Although our model derives in part from Malchow-Møller and Thorsen (2005), they examine optimal technology adoption and we explore the value anomaly.

We argue that cross-sectional differences in the number of growth options, growth rates, and investment costs may cause the observed value premium. When the ratio of the value of growth options to the value of assets-in-place becomes larger, the firm's growth options become more risky. This is particularly true for value firms since they wait longer before investing.[4] Alternatively, according to Cooper (2006), distressed firms have fewer growth options and experience idle capacity, causing their book-to-market equity ratios to rise. Therefore, value firms exhibit greater risk (than growth firms) attributable to their greater sensitivity to market conditions.

Our simulations indicate that growth firms: (1) invest sooner, (2) exhibit greater continuity in investment over time, (3) have lower book-to-market equity ratios, and (4) produce lower stock returns than value firms.[5] Our results suggest that, relative to value firms, the investment process for growth firms is more rapid and continuous, and that the value-growth stock effect may be a natural consequence of differences in growth options, investment irreversibility and cash flow risk.

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[4] We thank an anonymous reviewer for making this suggestion.

[5] Our results are consistent with those of Anderson and Garcia-Feijóo (2006) who find that growth (value) firms accelerate (slow) investment prior to portfolio formation year.

## 2. The model

Consider an all-equity firm with assets-in-place and growth option(s). The evolution of assets-in-place and of assets underlying growth options is given by:[6]

$$dA = \alpha_A A dt + \sigma_A A dz_A \tag{1}$$

and

$$dG = \alpha_G G dt + \sigma_G G dz_G, \tag{2}$$

where $A$ represents the present value of the expected cash flows generated by the firm's assets-in-place, $\alpha_A$ is the drift parameter for $A$, $\sigma_A$ is the constant volatility rate of the assets-in-place, and $z_A$ is a Wiener process.[7] Parameters in Equation (2) are defined similarly. $G$ represents the present value of expected cash flows from assets underlying the growth opportunities (and revised expected cash flows from assets-in-place). Both processes have increments of correlated Brownian motions, where $E(dz_A dz_G) = \rho dt$ with $-1 \leq \rho \leq 1$. To obtain a new cash flow generator, $G$, the firm must pay a fixed proportion, $i$, of $G$.[8] Consistent with Carlson, Fisher, and Giammarino (2004), we define cost $iG$ to encompass all costs associated with pursuing the growth opportunities (e.g., adjustment costs as well as the cost of new capital).

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[6] We define our diffusion processes, equations (1) and (2), to match Childs, Mauer, and Ott (2005), who use these diffusion processes for the present values of expected cash flows generated by assets-in-place ($A$) and by assets underlying the growth option ($G$) to study agency conflicts. We then adapt the Childs, Mauer, Ott diffusion processes to the repeated and single real options models of Malchow-Møller and Thorsen (2005). The latter paper uses repeated and single real options models to describe the levels of productivity of installed technology and exogenous technology. The levels of productivity of installed technology and exogenous technology in Malchow-Møller and Thorsen are analogous to the levels of cash flows in Childs, Mauer, and Ott. We adjust the Malchow-Møller and Thorsen models accordingly to make them compatible with the Childs, Mauer, and Ott diffusion processes.

[7] Our drift parameter, $\alpha$, is assumed to account for fixed depreciation per unit of time.

[8] Adopting $G$ does not necessarily mean that the firm totally changes its production process. For example, the firm can merely adopt new production technology to run alongside existing production processes, or may merely experience a management reorganization unrelated to production technology. The key points are that the firm has a new cash flow generator, and must pay a fixed proportion, $i$, of the value of the underlying assets of the growth option.

Following Malchow-Møller and Thorsen (2005), we specify that optimal policies do not depend explicitly on time. This eliminates the time derivative, leaving the following value function:[9]

$$V(G,A) = \max\{(r-\alpha_A)Adt + (1+rdt)^{-1}E[V(G+dG, A+dA \mid G, A],$$

$$(r-\alpha_A)Gdt - iG + (1+rdt)^{-1}E[V(G+dG, G+dA) \mid G, A=G]\}, \quad (3)$$

where $r$ is the discount rate. Invoking Ito's lemma, the continuation region of an unlevered firm is given by the partial differential equation:

$$\frac{1}{2}\sigma_A{}^2 A^2 V_{AA}^{"} + \frac{1}{2}\sigma_G{}^2 G^2 V_{GG}^{"} + \rho AG\sigma_A\sigma_G V_{AG}^{"} + \alpha_A AV_A^{'} + \alpha_G GV_G^{'} - rV + (r-\alpha_A)A = 0. \quad (4)$$

Assuming $V(G, A)$ is homogeneous, the normalized value of the firm can be written as:

$$v(w) = A^{-1}V(G,A), \quad (5)$$

where $w = G/A$. Following standard arguments (see Dixit and Pindyck, 1994, and Malchow-Møller and Thorsen, 2005), Equation (4) reduces to the following ordinary differential equation:

$$v(r-\alpha_A) + w(\alpha_A - \alpha_G)v' + \frac{1}{2}w^2(2\rho\sigma_A\sigma_G - \sigma_A^2 - \sigma_G^2)v" = r - \alpha_A, \quad (6)$$

having the general solution:

$$v = A_1 w^{a_1} + A_2 w^{a_2} + K, \quad (7)$$

where $a_1$, $a_2$, $A_1$, $A_2$ and $K$ are constants. Substituting Equation (7) into Equation (6) and appealing to standard arguments, firm value is given by:

$$v = A_1 w^{a_1} + 1, \quad (8)$$

where

[9] The specification of the value function follows from the infinite horizon models described in Dixit and Pindyck (1994). The term ahead of $Gdt$ is $r$-$\alpha_A$ instead of $r$-$\alpha_G$ because when $G$ is adopted, it inherits the properties of $A$. We thank one of the referees for making this suggestion.

$$a_1 = \frac{1}{2} - \frac{\alpha_G - \alpha_A}{\sigma_A^2 + \sigma_G^2 - 2\rho\sigma_A\sigma_G} + \sqrt{\left(\frac{1}{2} - \frac{\alpha_G - \alpha_A}{\sigma_A^2 + \sigma_G^2 - 2\rho\sigma_A\sigma_G}\right)^2 + \frac{2(r - \alpha_A)}{\sigma_A^2 + \sigma_G^2 - 2\rho\sigma_A\sigma_G}} > 1. \quad (9)$$

*2.1 Growth firms*

Brealey, Myers, and Allen (2008; pp. 98-101) describe growth firms as having a significant portion (i.e., a majority) of their value accounted for by the present value of growth opportunities (PVGO). These growth opportunities are valuable real options to expand, or to develop new products, processes, or technologies. Growth stocks sell at high price-earnings ratios because investors are willing to pay now in anticipation of outstanding performance on investments not yet made. As a result, growth firms tend to be characterized by low book to market equity ratios. Cooper (2006) asserts that these firms must undertake costly investment in order to benefit from positive economic circumstances.

In consideration of the above, we model the growth firm's plentiful investment opportunities as an infinite series of repeated real options. Growth firms expand according to rational rules of value maximization. If the value of the assets underlying the current growth option reaches $\lambda A$, the firm invests an amount $iG$, where $\lambda$ is the value of $G/A$ at the exercise boundary. The firm invests again when the value of assets underlying the next growth option, $G$, reaches the optimal exercise boundary, $\lambda^2 A$. By the Law of Motion (i.e., the "Clean Surplus" relation in accounting) for capital stock, investment cost $iG$ is accumulatively recorded as book equity *BE* (see Cooper, 2006). The value matching condition for this repeated option is:

$$v(\lambda) = \lambda v(1) - i\lambda . \quad (10)$$

Equation (10) takes the explicit form $V(G,A) = V(G,G) - iG$ at the optimal exercise.[10]

When the growth firm invests, $A$ is replaced by $G$, and investment cost $iG$ is deducted from firm value. Applying value matching and smooth pasting conditions to the general solution (Equation 8), we obtain the following expression for the normalized value of the growth firm:

$$V(G,A) = \frac{\lambda^{-a_1}}{a_1 - 1} G^{a_1} A^{1-a_1} + A, \tag{11}$$

and the following implicit trigger point function:

$$(1-i)(1-a_1) = \lambda^{-a_1} - a_1 \lambda^{-1}. \tag{12}$$

Intuitively, we would expect a firm whose investment opportunities are plentiful and available into the indefinite future to invest with some frequency. This intuition is supported by Malchow-Møller and Thorsen (2005) who show that in this setup, the expectation of cash flow from future investment options imposes a cost to keeping the current option alive. They show that this cost is impounded in equation (12).[11]

*2.2. Value firms*

Brealey, Myers, and Allen (2008) characterize value firms as having relatively few growth opportunities, with cash flow from current operations making up the largest proportion of share value. As examples, they state (p. 101), “Almost everyone regards…mature firms like Cummins or Dow Chemical as income stocks.” These firms would naturally have smaller price-earnings ratios because more of their value is tied up in existing operations. Higher *BE/ME* ratios should likewise follow. Cooper (2006) describes high *BE/ME* firms as having excess capital capacity, perhaps due to distress. These firms stand to benefit from a positive economic shock by virtue of the existence of this excess capital capacity. Regardless of the circumstances

[10] The explicit form is derived by substituting $v=V/A$ and $\lambda = G/A$ and making use of the homogenous property for $G/A$ and $V$.
[11] Derivations of Equations (11) and (12) are available from the authors.

surrounding the firm's current excess capital capacity, value firms are by nature less inclined to invest. Intuitively, we would expect a certain lumpiness to be a characteristic of their investment history.

As a result of the above considerations, we model a "typical" value firm as having a single investment opportunity. While this characterization is a highly stylized one, we place some importance on the firm's ability to make capital investment, albeit modest. The firm's optimal policy is to invest by paying $iG$ when $G$ reaches $\lambda A$ from below. Investment cost $iG$ will be recorded only once as book equity *BE*. The value matching condition for the value firm is:

$$v(\lambda) = \lambda - i\lambda \,. \tag{13}$$

Equation (13) takes the explicit form $V(G,A) = G - iG$ at the optimal exercise boundary. Value firms in our framework have a single growth option. Therefore, we apply value matching and smooth pasting conditions to the general solution (Equation 8), yielding the following expression for the normalized value of the value firm:

$$V(G,A) = \left((1-i)/a_1\lambda^{a_1-1}\right)G^{a_1}A^{1-a_1} + A\,, \tag{14}$$

and the following implicit trigger point function:

$$\lambda = \frac{1}{(1-\frac{1}{a_1})(1-i)}\,. \tag{15}$$

Intuitively, we would expect a firm with limited investment opportunities to delay investment significantly because the option to invest, once exercised, is gone forever. In contrast to our growth firm, Malchow-Møller and Thorsen (2005) show that there is no future investment opportunity to impose costs to keeping the investment option alive. Absent these costs, our value firm should delay investment relative to our growth firm.[12]

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[12] Derivation of Equations (14) and (15) are available from the authors.

*2.3. Optimal investment policies*

By substituting $\lambda = G/A$ into Equation (15), the value firm trigger point function may be re-expressed as

$$A + \frac{1}{a_1} G(1-i) = G(1-i)\,. \tag{16}$$

The left-hand side of Equation (16) is the value of the live growth option, and the right-hand side is the cost of keeping it alive. If the firm retains the option, it expects to receive the cash flows from the assets-in-place, *A*, plus an "adjusted value" offered by the option, $[1/a_1]G(1\text{-}i)$. The term $1/a_1$ adjusts for uncertainty ($a_1 > 1$; Equation 9) associated with option exercise. Therefore, $[1/a_1]G(1\text{-}i) < G(1\text{-}i)$, and

$$\left| \frac{\partial [A + (1/a_1) G(1-i)]}{\partial i} \right| \quad < \quad \left| \frac{\partial [G(1-i)]}{\partial i} \right|, \tag{17}$$

implying that a larger *i* reduces the benefit of option retention to a smaller degree than the corresponding cost reduction. The net effect is that firms delay investment because the benefit of retention becomes relatively greater than the corresponding cost. Similarly, since $\frac{\partial a_1}{\partial \alpha} < 0$, $\frac{\partial a_1}{\partial \sigma} < 0$, $\frac{\partial a_1}{\partial r} > 0$, and $\frac{\partial a_1}{\partial \rho} > 0$ an increase in $\alpha$ or $\sigma$ and a decrease in *r* or $\rho$ increases the benefit of holding the option, inducing the firm to wait longer.

The growth firm trigger function (Equation 12) may be expressed as:

$$A + \frac{1}{a_1} G(1-i) = G(1-i) + \frac{1}{a_1} G^{1-a_1} A^{a_1}, \tag{18}$$

where

$$\frac{1}{a_1} G^{1-a_1} A^{a_1} = \left(1 - \frac{1}{a_1}\right) \left[ V(G,G) - G \right], \tag{19}$$

and $[V(G,G) - G]$ is the difference in the trigger value between growth and value firms. For a given $A$, $G$ in Equation (16) will be greater than $G$ in Equation (18) because the second term on the right-hand side of Equation (18) is always positive. This implies that growth firms invest sooner than value firms. Changes in parameters $i$, $\alpha$, $\sigma$, $r$, and $\rho$ do not have identical effects on growth versus value firms.

## 3. A numerical illustration

Before discussing the simulation results, we illustrate our model with a simple numerical example. We derive values of the parameters that produce *BE/ME* differences between growth and value firms that are similar to those observed in historical U.S. data over 1977-2005.[13] The numerical illustration covers only the period through the first trigger point. Both firms are assumed to have identical starting points (for both firms, initial $G = A = 1$).

Table 1 applies the base case parameter values for Equations (11) and (14) to decompose firm value ($V$) into the present value of growth opportunities (*PVGO*) and the present value of expected cash flows generated by assets in place ($A$).[14]

---Please place Table 1 here ---

The growth firm invests sooner than the value firm, optimally investing when $G$ reaches $2.229A$, whereas the value firm waits until $G$ reaches $8.812A$. Because our numerical illustration initially sets $A$ equal to 1 for both firms, our findings also indicate that the dollar investment, each time the firm invests, decreases when the firm can invest more often. For example, the

[13] The average *BE/ME* for sampled firms (using COMPUSTAT data) over 1977-2005 equals 0.320 for the lowest 30 percent *BE/ME* group and equals 1.472 for the highest 30 percent *BE/ME* group, after winsorizing at the 1st and 99th percentiles. We solve for the value and growth firm alphas and investment costs that are consistent with these *BE/ME* differences.

[14] For our purposes, we define *PVGO* as the difference between firm value and present value of expected cash flows from assets-in-place.

growth firm spends 0.892 (equal to 2.23 x 0.40) while the value firm spends 7.49 (equal to 8.81 x 0.85), further highlighting the greater continuity of investment for growth firms.[15]

The *BE/ME* ratio changes substantially toward the time of investment, especially for the value firm. In Panel A, *BE*/*ME* increases from 0.317 to 0.932 for the growth firm, and increases from 0.699 to 6.310 for the value firm. *BE* rises abruptly, but *ME* rises less (see Equations 11 and 14).[16]

Summarizing, applying the empirically based parameters to our model, Table 1 shows that: (1) the optimal trigger point occurs sooner for growth firms than for value firms, (2) growth firms exhibit greater continuity in investment over time, and (3) *BE*/*ME* increases for investing firms.

Table 2 presents comparative statics results for various growth rate and investment cost assumptions. We consider 50 percent changes in $\alpha$ and 10 percent changes in $i$ from the base values. Equation (17) predicts that firms will postpone investment as either $\alpha$ or $i$ increase. To illustrate these predictions, we derive numerical illustrations for different $\alpha$ and $i$, centered around the value and growth firm base case parameters. Panels A and C illustrate how the optimal trigger point rises as growth rates rise, confirming the prediction of Equation (17). Both firms postpone investment and exhibit less continuity in investment when cash flow growth is high (i.e., $\lambda$ increases with $\alpha$ for both the growth and the value firm). Panels B and D illustrate how trigger points rise as investment costs rise. This result confirms the prediction of Equation (17) that, holding $\alpha$ constant, a larger value for $i$ reduces the benefit of option retention to a

[15] Lumpiness of investment is prevalent in models where firms face adjustment costs (e.g., Carlson, Fisher, and Giammarino, 2004; Cooper, 2006). However, in our setting, the adjustment costs are not explicitly separated from investment costs.

[16] The *BE/ME* ratio is likely to change significantly because firm value is continuous in *G/A* while investment cost is discontinuous.

smaller degree than the corresponding cost reduction, which leads to a delay in investment. The effects of 10 percent changes in *i* are much larger for the value firm.

--- Please place Table 2 here ---

## 4. Simulation analysis

We perform 100 simulations for 2,000 value firms and 2,000 growth firms over a 50 year period. Therefore, for each simulation, data are simulated for 100,000 value firm-years, and 100,000 growth firm-years. We simulate the present value of expected cash-flows for assets in place (*A*) for each firm using the general solution of the diffusion process in Equation (1):

$$A_{t_{i+1}} = A_{t_i} \exp\left( (\alpha_A - \frac{\sigma_A^2}{2})(t_{i+1} - t_i) + \sigma_A \sqrt{t_{i+1} - t_i} Z_A \right), \tag{20}$$

where *Z* is a standard normal random variable. The time series of the present value of expected cash-flows for assets underlying growth options (*G*) are derived using the Lévy representation of geometric Brownian motion.[17]

We calculate the value of both firm types using Equations (11) and (14) in which the book equity initially is set equal to *A* times proportional investment cost *i*. At a trigger point, the new investment cost, *iG*, is accumulated into the previous book equity. After calculating firm values, the value of *A* is replaced by its corresponding *G*. This happens once for value firms, and repeatedly for growth firms.

### *4.1. Book-to-market evolution*

[17] Given the processes, $dA = \alpha_A Adt + \sigma_A Adz_A$ and $dG = \alpha_G Gdt + \sigma_G Gdz_G$ with correlated Brownian motions $z_A$ and $z_G$ such that $E(dz_A\, dz_G) = \rho dt$, it may be shown that $z_G = \rho z_A + \sqrt{1-\rho^2} z_{A'}$ where $z_A$ and $z_{A'}$ are independent Brownian motions. From Levy's Theorem, $z_G$ is, in fact, a Brownian motion, and $E(dz_A\, dz_G) = \rho dt$. This is a simple method to simulate correlated Brownian motions by drawing from two independent standard normal distributions. For more details, see Shreve (2004), p. 171.

We follow procedures patterned after Cooper (2006) in which data are simulated for 2,000 growth firms and 2,000 value firms over 50 years (100,000 growth firm-years and 100,000 value firm-years, per simulation)   Following Cooper, we discard the first 20 years in order to generate stable cross-sectional data. Within each simulation, each year, the cross-sectional average value firm *BE/ME* ratio and growth firm *BE/ME* ratio are derived. This process is repeated for 30 years, yielding two sets of 30 cross-sectional average annual *BE/ME* ratios, for growth and value firms, respectively.

*4.2. Annual capital investment and passage time*

For each simulation, we derive annual capital investment, calculated as *CAPEX* = *iG*, for 2,000 value firms and 2,000 growth firms over 30 years (e.g., 60,000 firm-year *CAPEX* observations for value and growth firms, respectively) from a 50-year simulation discarding the first 20 years. As explained in Table 1, *G* initially equals 1 for all firms.

*4.3. Stock returns*

Following Cooper (2006), we simulate stock returns over a 600 month period.[18]  To be consistent with prior research that uses monthly data for returns analysis, we convert our base case parameters to monthly values. The cum-dividend realized rate of return for a firm in month *t* is

$$\frac{(V_t - \delta_A A_t) - (V_{t-1} - \delta_A A_{t-1}) + \delta_A A_t}{V_{t-1} - \delta_A A_{t-1}}, \tag{21}$$

where $\delta_A$ is dividend rate, equal to $r - \alpha_A$.

Individual stock (pre-ranking) betas are estimated using the 60-month moving window beginning in month 141 (within the total 600 month period). Using data for the remaining 400

[18] We use a risk-free rate of 0.15 percent per month and we use the market-value weighted average return of all firms to proxy the market portfolio return. Similarly, in his simulation, Cooper (2006) uses an annual risk-free rate of 1.8%.

months, we sort firms into size deciles and then into beta deciles to obtain a total of 100 dependent-sort (equally-weighted) portfolios. Each portfolio comprises 20 stocks. We run time-series regressions over the remaining 400 month period to derive post-ranking beta estimates for the 100 portfolios. Stocks are assigned a beta equal to the post-ranking beta for the portfolio to which they are assigned. For each month in the 400 month performance period, cross-sectional regressions are run to obtain slopes associated with *beta* and *ln*(*BE/ME*):

$$R_{it} = \gamma_0 + \gamma_1 beta_i + \gamma_2 ln(BE/ME)_{i,\,t-1} + \varepsilon_{it}, \tag{22}$$

where $beta_i$ is the post-ranking beta assigned to stock *i* and $ln(BE/ME)_{i,t-1}$ is the natural logarithm for the lagged *BE*/*ME* ratio for firm *i*.[19] In the Appendix, we show how beta is explicitly related to the modeled parameters.

We calculate the average the monthly slopes across the 400 cross-sectional regressions. The *t*-statistics are the averaged slope estimates divided by the corresponding standard error.[20] Once again, we run 100 simulations. To ensure equal representation of value and growth firms in the simulations, half of the firms are value firms and half of the firms are growth firms.

*4.4 Results*

Table 3 presents comparative static results of 100 simulations for *BE/ME* for the six years leading up to and including the portfolio formation year, after altering the growth rate $\alpha$, and investment cost *i*. We note that the formation year (year *t*) is not selected based on hitting an investment trigger. Firms might hit triggers throughout the period. Each growth firm might invest multiple times, whereas each value firm invests just once at most. We restrict the

[19] Since the Brownian motion of the present value of expected cash flows of assets-in-place and growth options is, on average, exponentially increasing over time as shown in Equation (20), firm size will be positively correlated with firm's realized return in our model (as a consequence of the Brownian motion assumption). Therefore, we do not include *SIZE* in our tests.

[20] We also perform robustness tests to confirm that the value stock return effect still exists even after varying the parameters from their base case settings. Primary results are unchanged.

examination of *BE/ME* ratios to the 6 years leading up to and including the year of stock classification. We do so in order to compare against empirical data showing that *BE/ME* ratios trend downward for growth firms and trend upward (indicative of rising distress risk) for value firms in the years prior to portfolio formation.

The simulated patterns are consistent with extant empirical findings. *BE/ME* ratios trend upward for value firms (and trend downward for growth firms), consistent with the distress rationale offered by Fama and French (1995). Also consistent with empirical findings, data reported in rows 2 and 3 of either panel indicate that lower growth ($\alpha$) is associated with higher *BE/ME* (e.g. Lakonishok, Shleifer and Vishny, 1994, Anderson and Garcia-Feijóo, 2006, and Cooper, Gulen, and Schill, 2008).[21] Moreover, lower investment cost (row 4 versus row 5 of either Panel) is associated with lower *BE/ME*. This result suggests that the difference between value and growth firms may be modeled as the size of the investment cost relative to the investment's value.[22]

Figures 1 and 2 illustrate the distribution of capital expenditures from simulated growth firms and value firms, respectively. The figures scale the capital expenditures by the overall mean value calculated over all simulations for both growth and value firms (e.g., a "relative investment" equals 1 for a capital expenditure that equals the overall average). We find that the mean capital expenditure is significantly larger across the 100 simulations for the value firms than for the growth firms (*t*-statistic for the difference is 8.72). In addition to the difference in the

[21] We also run multiple simulations to decompose the difference in *BE/MEs* between growth and value firms into the proportions attributable to differences in the options structure in our model versus the use of different parameters. We ran simulations for growth firms with growth (high) alphas, growth firms with value firm (low) alphas, and value firms with value (low) alphas. We find that approximately 30% of the difference in *BE/MEs* in our simulations is attributable to the use of different alpha parameters. The remaining 70% of the difference is attributable to differences in the options structure in our model.

[22] We thank an anonymous referee for this suggestion.

means, we observe that the standard deviation of capital expenditure is smaller for growth firms, indicating greater continuity in the capital expenditure path.[23]

Figures 3 and 4 show the distribution of investment passage times from simulated growth firms and value firms, respectively. We convert the timeline into percentage time units in order to provide easier comparison in the investment passage time exhibited by our simulated value and growth firms. Consistent with our capital expenditure distributions, growth firm passage times are shorter with less variation than value firm passage times. We note (details not reported) that the difference in passage times between growth and value firms becomes more dramatic if we allow $\alpha_G$ to exceed $\alpha_A$. Results shown in Figures 1 through 4 are consistent with the prediction that growth firms invest more quickly and with greater continuity over time than value firms.

We also perform an empirical analysis to compare our results to historical U.S. data over 1977-2005. We find that mean capital expenditures are significantly higher for value firms (highest *BE/ME* quintile firms) versus growth firms (lowest *BE/ME* quintile firms). These results hold within every firm size quintile (analogous to setting $G = 1$ for every firm in our simulation). We also find that the standard deviation and skewness of capital expenditures are higher for the portfolio of value firms. In addition, means and medians of capital expenditure growth rates are higher for growth firms in all firm size groups, indicating that growth firms invest faster than value firms. These results are consistent with our simulation, as well as with Anderson and Garcia-Feijóo (2006), who show that higher capital expenditure growth rates are associated with shorter investment passage times.

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[23] In our simulation, the mean capital expenditure is nearly 50% larger and the standard deviation is approximately 4.75 times larger for value firms versus growth firms.

In our cross-sectional stock return examination, we estimate Equation (22) using stock returns simulated from our model and the parameter settings shown in the first rows of Panel A and B of Table 3. For each simulation, we derive the time-series average risk premiums across 400 regressions. Also, for each simulation, we derive the *t*-statistic for each average risk premium, then calculate the average risk premiums and average *t*-statistics across the 100 simulations.

We find that the risk premiums for *beta* and *BE/ME* are positive and statistically significant: $\hat{\gamma}_1$ = 0.6315 percent and $\hat{\gamma}_2$ = 0.1851 percent, with *t*-statistics of 13.18 and 11.71 and respectively.[24] Therefore, findings from our simulation indicate that both beta and *BE/ME* are priced, and that stocks with high *BE/ME* ratios are associated with higher stock returns, after controlling for beta risk. Overall, the results indicate that real options models that incorporate investment irreversibility and cash flow risk are sufficient to produce the value stock return effect.

## 5. Conclusions

We use a real options model to provide insight into the value stock return anomaly. Firm value evolves in response to optimal investment decisions. These investments generate new cash flows from assets underlying growth options. Our simple model identifies two types of firms. One type possesses a single investment opportunity, and we identify it as a "value stock" firm. The other type possesses an infinite series of investment opportunities, and we identify it as a

[24] Our beta premium results are similar to those reported using ex-ante expectational data by Brav, Lehavy, and Michaely (2005). Moreover, many researchers find significant and positive beta premiums over specific time periods (dating back to Fama and MacBeth, 1973, and Black, Jensen, and Scholes, 1972). Of course, not all tests find significant beta premiums (Fama and French, 1992). Our *BE/ME* premium results agree with produced by Cooper's (2006) real options model. Cooper's source of uncertainty is measured by the ratio of the firm's stock of capital to its productivity. This ratio, by construction, is positively correlated to the book-to-market ratio. Therefore, the source of uncertainty also is reflected in the book-to-market ratio itself, which would induce a significant *BE/ME* premium. Our model, however, is constructed using expected cash flows as the source of uncertainty, yet still produces a significant *BE/ME* premium.

“growth stock” firm. While we admit that the dichotomy is simplification on a grand scale, our simulation results suggest that the model is consistent with empirical data and extant literature

Our simulation shows that, relative to value firms, growth firms invest more quickly, exhibit greater continuity in investment, have lower book-to-market ratios, and produce lower rates of return. Our results also are consistent with the value stock return effect. Firms facing higher degrees of investment irreversibility (i.e., value firms) are likely to provide higher rates of return as compensation for risk.

**Appendix. Derivation of beta and its relation to the book-to-market ratio**

A closed form solution for beta obtains under an assumption about the diffusion coefficients for *A* and *w*. Specifically, this appendix shows that beta can be expressed as a closed-form solution of *G/A*, and depends on whether the firm has indeterminate options or a single one.

We begin with expressions for the normalized value of the firm (Equation (5)), and the cum-dividend return (Equation 21):

$$v(w) = A^{-1}V(G, A) \tag{5}$$

$$\frac{(V_t - \delta_A A_t) - (V_{t-1} - \delta_A A_{t-1}) + \delta_A A_t}{V_{t-1} - \delta_A A_{t-1}}. \tag{21}$$

The return may be re-expressed as:

$$\frac{d(vA) + \delta_A A}{vA - \delta_A A} \tag{A.1}$$

Rewriting equation (A.1.), we obtain:

$$\frac{dA}{A}\left(\frac{v}{(v-\delta_A)}\right) + \frac{dv}{v}\left(\frac{v}{(v-\delta_A)}\right) + \frac{\delta_A}{(v-\delta_A)}\ . \tag{A.2}$$

Application of Ito's Lemma yields the stochastic differential equation:

$$dv = [\alpha_w v' w + \frac{1}{2}\sigma_w v'' w^2]dt + v' w \sigma_w dz_w\ . \tag{A.3}$$

By definition, beta may be derived from:

$$\text{cov}\left(\frac{dV + \delta_A A}{V - \delta_A A}, R_m\right), \tag{A.4}$$

which equals:

$$\frac{v}{(v-\delta_A)}\left[\text{cov}\left(\sigma_A dz_A, R_m\right) + \text{cov}\left(\frac{v' w \sigma_w dz_w}{v}, R_m\right)\right]. \tag{A.5}$$

If we assume that $\sigma_A dz_A = \sigma_w dz_w = \sigma dz$, then we may rewrite beta as:

$$\operatorname{cov}\left(R, R_m\right) = \frac{v\sigma}{(v-\delta_A)}\left[\operatorname{cov}\left(dz, R_m\right)\left(1+\frac{v' w}{v}\right)\right], \tag{A.6}$$

which yields the following expression for beta:

$$\beta = \frac{\operatorname{cov}\left(R, R_m\right)}{\operatorname{var}\left(R_m\right)} = \frac{\sigma(v+v' w)}{(v-\delta_A)}\left[\frac{\operatorname{cov}\left(dz, R_m\right)}{\operatorname{var}\left(R_m\right)}\right] \tag{A.7}$$

We observe that beta is a function of $w = G/A$. When the ratio of the value of growth options to the value of assets in place becomes larger, the firm's growth options become riskier. This is especially true for value firms because they wait longer before investing.

We note that for growth firms:

$$v(w) = \left(\frac{\lambda^{-a_1}}{a_1 - 1}\right) w^{a_1} + 1\,, \tag{A.8}$$

and for value firms:

$$v(w) = \left((1-i)/\, a_1 \lambda^{a_1 - 1}\right) w^{a_1} + 1\,. \tag{A.9}$$

Table 1

**Effects of assets-in-place and growth options on firm decision, firm value, book equity, and book-to-market ratio**

Normalized values of the assets underlying the growth option (*G/A*), firm value, *PVGO*, present values of expected cash flows generated by the firm's assets-in-place (*A*), book equity, and book equity to market equity (*BE/ME*) ratios are presented for the hypothetical growth and value firm. Base case values of the parameters are chosen to produce *BE/ME* differences between value and growth firms similar to those observed in historical U.S. data over 1977-2005. Base case values are: initial value of assets in place (*A* ) = 1, initial value of the asset underlying the growth option (*G*) = 1, $\sigma_V = \sigma_G$ = 5.77% per month, $\alpha$ = 0.15% per month, and 0.38% per month for the value and growth firm, respectively, $i$ = 0.85 and 0.40 for the value firm and growth firm, respectively, and $r$ = 1% per month, and $\rho$ = 0.80. Initial book equity is proportional to the initial investment cost of assets-in-place, *iA*. Trigger points are reported in bold.

| Normalized value of the asset underlying the growth option (*w* = *G/A*) | Firm value | *PVGO* | *BE* | *BE/ME* |
|---|---|---|---|---|
| *Panel A. The growth firm* | | | | |
| 1.000 | 1.022 | 0.022 | 0.400 | 0.392 |
| 1.500 | 1.093 | 0.093 | 0.400 | 0.366 |
| 2.000 | 1.261 | 0.261 | 0.400 | 0.317 |
| **2.229** | **1.386** | **0.386** | **1.292** | **0.932** |
| *Panel B. The value firm* | | | | |
| 1.000 | 1.000 | 0.000 | 0.850 | 0.850 |
| 2.000 | 1.001 | 0.001 | 0.850 | 0.849 |
| 3.000 | 1.004 | 0.004 | 0.850 | 0.847 |
| 4.000 | 1.013 | 0.013 | 0.850 | 0.840 |
| 5.000 | 1.031 | 0.031 | 0.850 | 0.824 |
| 6.000 | 1.066 | 0.066 | 0.850 | 0.797 |
| 7.000 | 1.125 | 0.125 | 0.850 | 0.756 |
| 8.000 | 1.216 | 0.216 | 0.850 | 0.699 |
| **8.812** | **1.321** | **0.322** | **8.340** | **6.310** |

Table 2

**Sensitivity analysis for trigger points**

This table presents comparative statics results for different growth rate and investment cost assumptions. The analysis is centered around the α and *i* that reproduce the *BE/ME* differences between growth and value firms observed in historical data in the U.S. over 1977-2005.

| Drift parameter | Investment cost | Optimal trigger |
|---|---|---|
| *Panel A. Sensitivity analysis for α for the growth firm* | | |
| $\alpha = 0.0019$ | $i = 0.40$ | 2.1645 |
| $\alpha = 0.0038$ | $i = 0.40$ | 2.2291 |
| $\alpha = 0.0057$ | $i = 0.40$ | 2.3277 |
| *Panel B. Sensitivity analysis for i for the growth firm* | | |
| $\alpha = 0.0038$ | $i = 0.36$ | 2.0743 |
| $\alpha = 0.0038$ | $i = 0.40$ | 2.2291 |
| $\alpha = 0.0038$ | $i = 0.44$ | 2.4037 |
| *Panel C. Sensitivity analysis for α for the value firm* | | |
| $\alpha = 0.00075$ | $i = 0.850$ | 8.7113 |
| $\alpha = 0.00150$ | $i = 0.850$ | 8.8119 |
| $\alpha = 0.00225$ | $i = 0.850$ | 8.9280 |
| *Panel D. Sensitivity Analysis for i for the value firm* | | |
| $\alpha = 0.00150$ | $i = 0.765$ | 5.6246 |
| $\alpha = 0.00150$ | $i = 0.850$ | 8.8119 |
| $\alpha = 0.00150$ | $i = 0.935$ | 20.3351 |

Table 3

**Simulation of the evolution of book-to-market ratio**

Within one simulation, each year, annual *BE/ME* ratios are averaged separately across value and growth firms. This process is repeated for 30 years, producing 30 annual *BE/ME* ratios for growth and value firms. These *BE/ME*s are then averaged separately for value and growth firms. Comparative statics results are reported for *BE/ME* for the six years leading up to and including the portfolio formation year, after altering the growth rate $\alpha$ and investment cost $i$. A total of 100 simulations are run.

| Parameter setting | | | Average *BE/ME* | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\alpha$ | $\sigma$ | $i$ | $t$-5 | $t$-4 | $t$-3 | $t$-2 | $t$-1 | $t$ |
| *Panel A. Value firms* | | | | | | | | |
| 0.018 | 0.200 | 0.850 | 1.6905 | 1.7101 | 1.7297 | 1.7496 | 1.7697 | 1.7901 |
| 0.009 | 0.200 | 0.850 | 2.2447 | 2.2831 | 2.3222 | 2.3624 | 2.4031 | 2.4445 |
| 0.027 | 0.200 | 0.850 | 1.2723 | 1.2806 | 1.2888 | 1.2971 | 1.3053 | 1.3137 |
| 0.018 | 0.200 | 0.750 | 1.4498 | 1.4652 | 1.4808 | 1.4967 | 1.5127 | 1.5288 |
| 0.018 | 0.200 | 0.950 | 1.8892 | 1.9122 | 1.9356 | 1.9595 | 1.9837 | 2.0081 |
| *Panel B. Growth firms* | | | | | | | | |
| 0.045 | 0.200 | 0.400 | 0.3441 | 0.3436 | 0.3431 | 0.3426 | 0.3421 | 0.3416 |
| 0.027 | 0.200 | 0.400 | 0.5721 | 0.5752 | 0.5784 | 0.5815 | 0.5847 | 0.5879 |
| 0.063 | 0.200 | 0.400 | 0.2148 | 0.2133 | 0.2119 | 0.2105 | 0.2091 | 0.2077 |
| 0.045 | 0.200 | 0.300 | 0.2756 | 0.2754 | 0.2751 | 0.2748 | 0.2746 | 0.2743 |
| 0.045 | 0.200 | 0.500 | 0.4173 | 0.4165 | 0.4156 | 0.4147 | 0.4139 | 0.4130 |

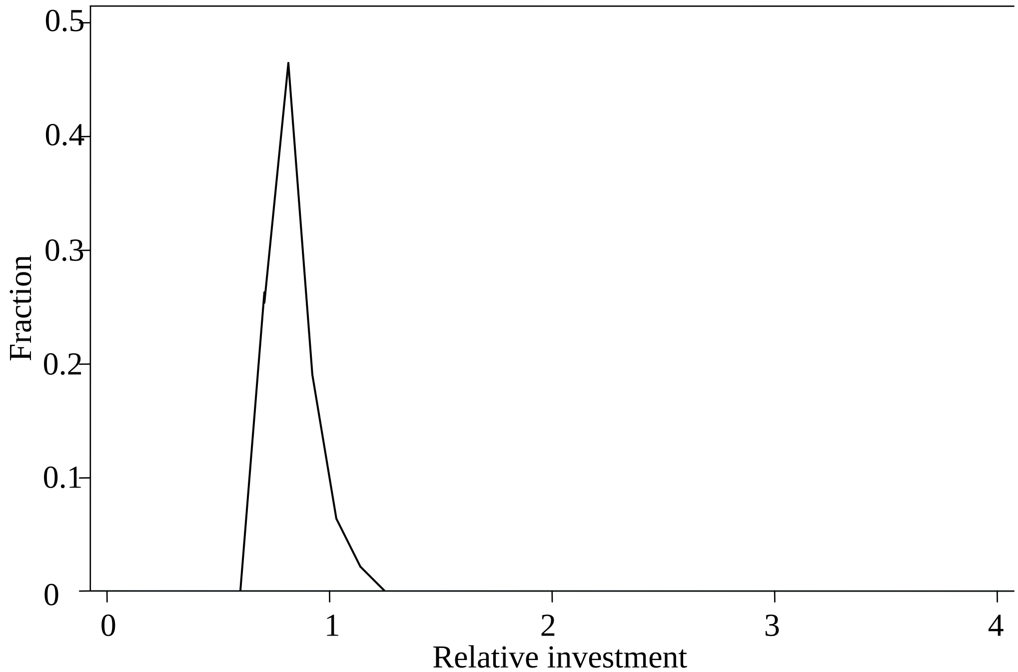


Figure 1
**Average relative capital expenditures from 100 growth firm simulations**
($\alpha$ = 0.045; $\sigma$ = 0.200; $i$ = 0.40)

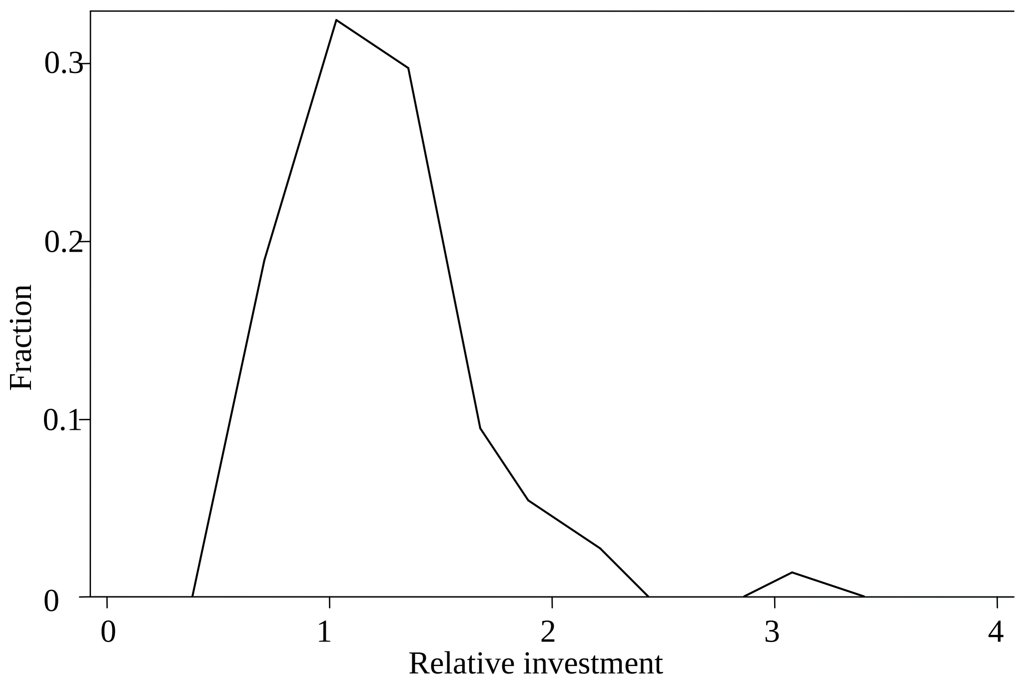


Figure 2

**Average relative capital expenditures from 100 value firm simulations**

($\alpha$ = 0.018; $\sigma$ = 0.200; $i$ = 0.85)

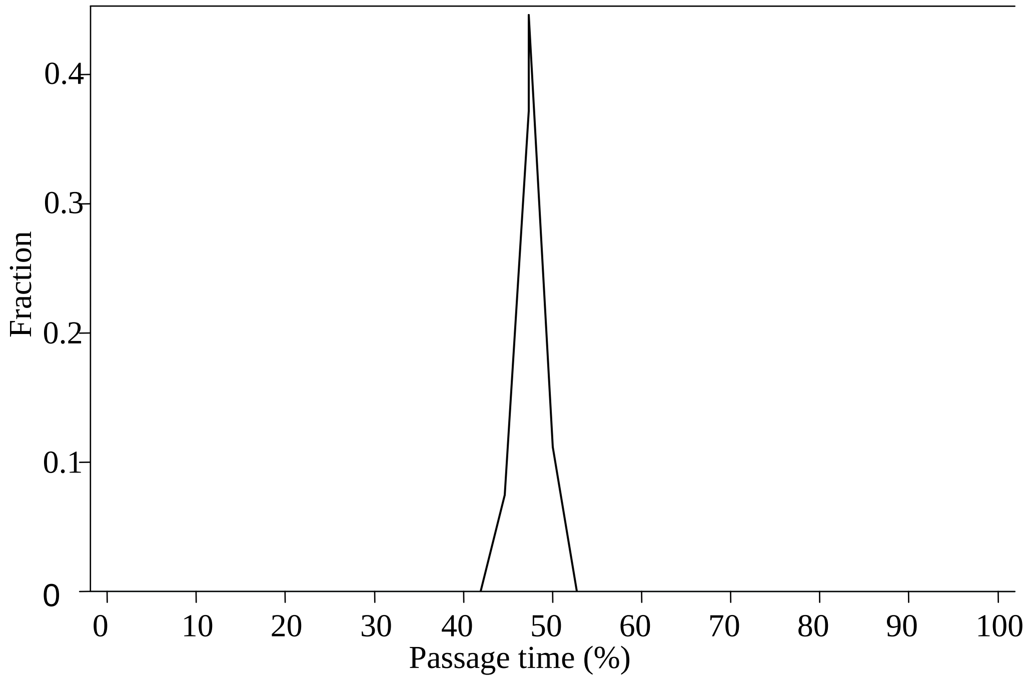


Figure 3
**Percentage of average passage time from 100 growth firm simulations**
($\alpha$ = 0.045; $\sigma$ = 0.200; $i$ = 0.40)

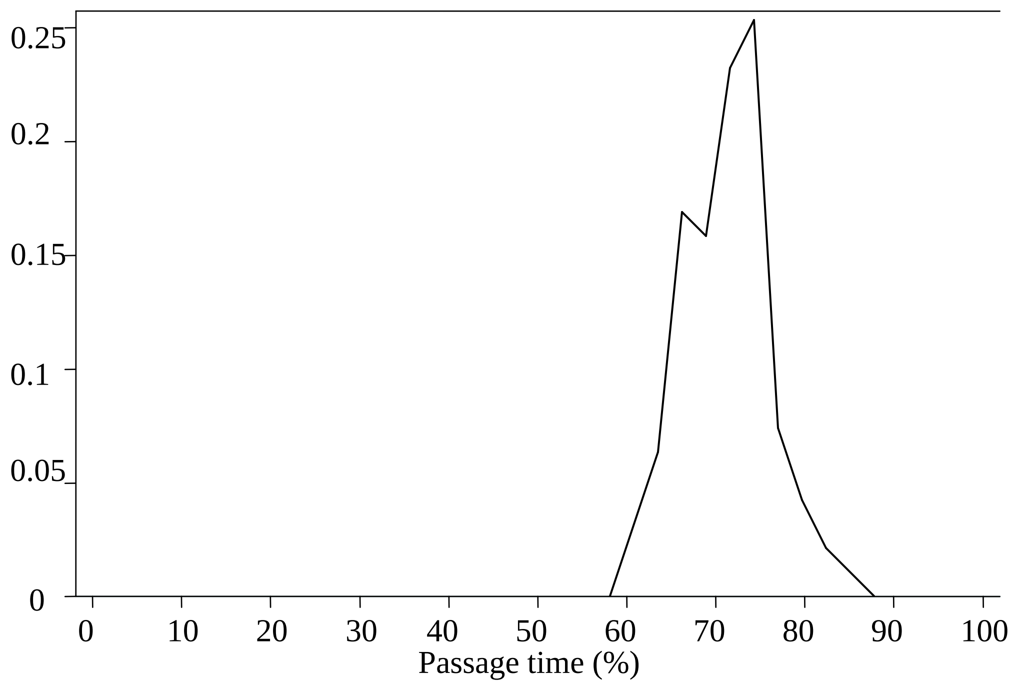


Figure 4
**Percentage of average passage time from 100 value firm simulations**
($\alpha$ = 0.018; $\sigma$ = 0.200; $i$ = 0.85)

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