ARBITRAGE AND THE EVALUATION OF LINEAR FACTOR MODELS IN U.K. STOCK RETURNS

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I examine the impact of the no arbitrage restriction on the estimation and evaluation of linear factor models in U.K. stock returns. The no arbitrage restriction reduces volatility and eliminates most of the negative values of the fitted stochastic discount factor models. All of the factor models are rejected and there are significant differences in the pricing performance between models under the no arbitrage restriction. The no arbitrage restriction can have a significant impact on both the parameter estimates and pricing errors for some models.

Key Words: Stochastic discount factor, no arbitrage, distance measures

JEL Classification: G12

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1. Introduction

The seminal study by Hansen and Jagannathan (1997) proposes two distance measures (denoted HJD) to evaluate misspecified asset pricing models that allows for model comparison. The second HJD takes into account the no arbitrage restriction when evaluating asset pricing models. A recent study by Li, Xu, and Zhang (2009) develops the asymptotic distribution theory of the second HJD under the null of a true model. Li, Xu, and Zhang also develop the distribution theory for the estimation of stochastic discount factor models using the second HJD. The no arbitrage restriction is an important issue in the estimation and evaluation of asset pricing models since the second HJD provides a more accurate evaluation of asset pricing models because it penalizes models that violate the no arbitrage requirement (e.g., Wang and Zhang, 2006; Li, Xu, and Zhang, 2009). Li, Xu, and Zhang also argue that estimating the model parameters for linear factor models to minimize the second HJD can lead to better proxies for marginal utility growth (e.g., Cochrane, 2005) and might improve the out-of-sample performance of such models.

Using the distribution theory of Li, Xu, and Zhang (2009), I examine the impact of the no arbitrage restriction in the estimation and evaluation of linear factor models in U.K. stock returns. The factor models considered are unconditional versions of the standard capital asset pricing model (CAPM), the CAPM model of Jagannathan and Wang (1996), an empirical model similar to Fama and French (1993), the intertemporal CAPM (ICAPM) (e.g., Merton, 1973), and the linear consumption CAPM (CCAPM). I also examine conditional versions of the CAPM, CCAPM, and the Fama and French model.

My study makes two main contributions to the literature. First, I extend the prior literature of asset pricing studies in U.K. stock returns and in other financial markets. Fletcher and Kihanda (2005), Hyde and Sherif (2005), Fletcher (2007), Gao and Huang (2008), among others, provide asset pricing evidence in U.K. stock returns. Recent studies by Engsted and Moller (2008) examine the performance of nonlinear CCAPM models in Denmark. Schrimpf, Schroder, and Stehle (2007) and Durack, Durand and Maller (2004) explore the performance of the conditional CAPM in German and Australian stock returns respectively. These latter studies all use the first HJD as one metric for evaluating model performance. I extend this literature by using and examining the importance of the no arbitrage restriction on the estimation and evaluation of linear factor models. Second, my study complements the recent studies of Wang and Zhang (2006), Li, Xu, and Zhang (2009), and Chen and Ludvigson (2009) by examining the importance of the second HJD in evaluating asset pricing models in a non-U.S. market.

There are three main findings in this paper. First, the no arbitrage restriction has a significant impact on the evaluation of linear factor models. There is evidence against all linear factor models using the second HJD. Second, the no arbitrage restriction has a significant impact on the estimation of linear factor models. Using the second HJD leads to a less volatile stochastic discount factor, reduces most of the negative stochastic discount factor values, and moderates the size and increases the statistical significance for a number of factors and scaled factors. Third, using the second HJD to estimate and evaluate linear factor models increases the magnitude and statistical significance of the pricing errors of the test assets for some models. I consider a conditional version of the Fama and French (1993) model as the best performing model. However, this model performs poorly in out-of-sample pricing tests. The findings of this paper highlight the importance of using the second HJD framework for estimating and evaluating asset pricing models.

2. Research method

Ross (1978), Harrison and Kreps (1979), Hansen and Richard (1987), among others, show that if the law of one price holds in financial markets, then there exists a stochastic discount factor m_t such that:

$$E_{t-1}(m_t x_{it}) = p_{it-1} \qquad \text{for } i = 1,\ldots,N \tag{1}$$

where x_{it} is the payoff on asset i at time t, p_{it-1} is the price of asset i at time t-1, and N is the number of primitive assets. When financial markets satisfy the no arbitrage condition, m_t will be positive in every state of nature (e,g., Cochrane, 2005). The stochastic discount factor will only be unique if markets are complete. I focus on the unconditional implication of equation (1)[1] as:

$$E(m_t x_{it}) = E(p_{it-1}) \qquad \text{for } i = 1,\ldots,N \tag{2}$$

The difference between the left-hand and right-hand-sides of equation (2) is the pricing error of asset i.

Hansen and Jagannathan (1997) develop a framework to evaluate candidate stochastic discount factors for which equation (2) might not be true. Therefore, I define set M (M^+) as the set of stochastic discount factors (nonnegative stochastic discount factors) that price the assets under consideration. Hansen and Jagannathan (1997) propose two distance measures. The first HJD estimates the minimum least squares distance between a given candidate model (y_t) and M. The squared first HJD (d^2) is given by:

$$d^2 = \min\,(m \in M)\; E[(y_t - m_t)^2] \tag{3}$$

The first HJD is the largest pricing error of a portfolio of primitive assets with unit norm. The second HJD captures the least square distance between a given candidate model y_t and M^+. The squared second HJD (d^{+2}) is given by:

[1] I do not focus on incorporating the impact of conditioning information on the set of payoffs that the candidate stochastic discount factor models are required to price (e.g., Cochrane, 1996).

$$d^{+2} = \min\ (m \in M^+)\ E[(y_t - m_t)^2] \tag{4}$$

The second HJD is the minmax bound on the pricing errors of all potential derivative claims on the N primitive assets. The difference between the first and second HJD is that the set M can include admissible stochastic discount factors that are negative in certain states of the world. The set M^+ is smaller than the set M. As a result, the second HJD (d^+) will be greater than or equal to the first HJD (d).

The first and second HJD in equations (3) and (4) can be extended to allow the candidate stochastic discount factor models to have unknown parameters $d^2(\gamma)$ and $d^{+2}(\gamma^+)$ where $\gamma(\gamma^+)$ is the set of unknown parameters in the candidate model when the model parameters are estimated to minimize $d^2(d^{+2})$. Hansen and Jagannathan (1997) show that we can estimate $\gamma(\gamma^+)$ and $d(d^+)$ by solving the following conjugate problems:

$$d^2 = \min_\gamma \max_\lambda E_T\Phi(\gamma,\lambda) \tag{5}$$

$$d^{+2} = \min_\gamma \max_\lambda E_T\Phi^+(\gamma^+,\lambda^+) \tag{6}$$

where

$$E_T\Phi(\gamma,\lambda) = (1/T)\Sigma^T_{t=1} y_t(\gamma)^2 - (y_t(\gamma)\text{-}\lambda' x_t)^2 - 2\lambda' p_{t-1}$$

$$E_T\Phi^+(\gamma^+,\lambda^+) = (1/T)\Sigma^T_{t=1} y_t(\gamma^+)^2 - (y_t(\gamma^+)\text{-}\lambda^{+\prime} x_t)^{+2} - 2\lambda^{+\prime} p_{t-1}$$

$\lambda(\lambda^+)$ is a (N,1) vector of Lagrange multipliers; x_t is a (N,1) vector of payoffs at time t; p_{t-1} is a (N,1) vector of prices at time t-1; T is the number of observations; and $(y_t(\gamma^+)\text{-}\lambda^{+\prime} x_t)^+ = \max(0, y_t(\gamma^+)\text{-}\lambda^{+\prime} x_t)$. Define θ as (γ, λ) and θ^+ as (γ^+, λ^+). The $(y_t(\gamma^+)\text{-}\lambda^{+\prime} x_t)^+$ term is the payoff on an option that is a solution to equation (6).

The solutions to equations (5) and (6) imply that $(y_t(\gamma)\text{-}\lambda' x_t)$ is a stochastic discount factor that belongs to the set M, and $(y_t(\gamma)\text{-}\lambda^{+\prime} x_t)^+$ is a stochastic discount factor that belongs to set M^+. The λ multipliers that solve equation (5) are unique, and $\lambda' x_t$ is the minimum adjustment required to make the candidate model y_t belong to set M. The λ^+ multipliers that solve equation (6) are not unique but $(y_t(\gamma^+)-\lambda^{+\prime}x_t)^+$ is unique (e.g., Hansen, Heaton, and Luttmer, 1995; Hansen and Jagannathan, 1997). The term $y_t(\gamma^+)-(y_t(\gamma^+)-\lambda^{+\prime}x_t)^+$ is the minimum adjustment required to make the candidate model y_t to belong to set M^+.

In this study, the focus is on linear factor models as the stochastic discount factor models where K is the number of parameters in the model. Estimating the parameters to minimize the first HJD can be set within a Generalized Method of Moments (GMM) (e.g., Hansen, 1982) framework. Hodrick and Zhang (2001) provide an excellent overview of the approach. A recent study by Li, Xu, and Zhang (2009) derives the corresponding distribution theory for estimating the model parameters to minimize the second HJD that allows for potential model misspecification.[2] The challenge in deriving the asymptotic distribution of the second HJD is that the function $\Phi^+(\gamma^+,\lambda^+)$ is not differentiable everywhere since the second derivative does not exist when $y_t(\gamma^+)-\lambda^{+\prime}x_t = 0$. Li, Xu, and Zhang overcome this problem using the concept of "differentiation in quadratic mean" (e.g., Pakes and Pollard, 1989; Hansen, Heaton, and Luttmer, 1995).

Li, Xu, and Zhang (2009) develop their main results by deriving the second-order asymptotic representation of d^{+2} at the estimated θ^+ in Lemma 2 of their paper. Define $A = (A_{\gamma+}, A_{\lambda+})$ where $A_{\gamma+}$ and $A_{\lambda+}$ are the time-series processes of the first-order derivatives of Φ^+ with respect to γ^+ and λ^+. Li, Xu, and Zhang assume that the central limit theorem holds for A such that $T^{1/2}A$ has an asymptotic normal distribution as $N(0, \Lambda)$. The second-order partial derivatives of Φ^+ is given by Γ with the respective dimensions of Γ_{11} (K,K), Γ_{12} (K,N), Γ_{21} (N,K), and Γ_{22} (N,N) where 1(2) refers to differentiating Φ^+ with respect to $\gamma^+(\lambda^+)$. Li, Xu, and Zhang show that under

[2] Kan and Robotti (2009) derive the asymptotic distribution of the model parameters for a misspecified model using the first HJD.

the null hypothesis of the model having a zero second HJD[3], Td^{+2} has a weighted χ^2 distribution where the weights come from the eigenvalues of:

$$-1/2[\Gamma^{-1}_{22} - \Gamma^{-1}_{22}\Gamma_{21} (\Gamma_{12}\Gamma^{-1}_{22}\Gamma_{21})^{-1}\Gamma_{12}\Gamma^{-1}_{22}]\Lambda_{\lambda} \tag{7}$$

where Λ_{λ} is a (N,N) submatrix of Λ that applies to $A_{\lambda+}$. The test of a zero second HJD can be used in the same way as for the Jagannathan and Wang (1996) test of a zero first HJD.

Li, Xu, and Zhang (2009) also derive the asymptotic distribution of γ^+ and λ^+. The asymptotic distribution of γ^+ is given by:

$$N\ (0, (J_{11}\ J_{12})\Lambda(J_{11}\ J_{12})') \tag{8}$$

where $J= \Gamma^{-1}$ with $J_{11} = [\Gamma_{11} - \Gamma_{12}\Gamma^{-1}_{22}\Gamma_{21}]^{-1}$ and $J_{12}=-[\Gamma_{11} - \Gamma_{12}\Gamma^{-1}_{22}\Gamma_{21}]^{-1}\Gamma_{12}\Gamma^{-1}_{22}$. The distribution of λ^+ has the same format as (8) except J_{11} and J_{12} are replaced by J_{21} and J_{22}, which come from the relevant dimensions of J. The standard errors of γ^+ that come from (8) are valid whether the model is correctly specified or not. Li, Xu, and Zhang point out that the covariance matrix of γ^+ can be simplified under the null that the model is correctly specified. An earlier working paper version (2004) of Li, Xu, and Zhang also derives the asymptotic distribution of the pricing errors of the primitive assets using the second HJD under the null that the model is true, which I use to test the statistical significance of the pricing errors.

3. Data

The data come from the London Share Price Database (LSPD) at the London Business School, the U.K. Office for National Statistics, and the U.K. country data from the International Financial Statistics (IFS) of the International Monetary Fund. Details of the formation of the primitive assets, the factors in the linear factor models, and the information variables used in the conditional models are described in a separate appendix (available from the author).

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[3] Hansen, Heaton, and Luttmer (1995) derive the asymptotic distribution of the first and second HJD under the null that the model is false.

3.1. Primitive assets

I evaluate the pricing performance of the linear factor models in U.K. stock returns using the quarterly excess returns of sixteen portfolios of stocks sorted by size and dividend yield and the quarterly return of a three-month U.K. Treasury Bill as the set of primitive assets. I use the dividend yield to capture the value-growth effect rather than the book-to-market ratio due to data availability. Dimson, Nagel, and Quigley (2003) find that the value effect is not as strong with the dividend yield, but there is a high positive correlation between zero-cost portfolios formed using the dividend yield and book-to-market ratios. Chan, Karceski, and Lakonishok (1998) also find a dividend yield effect in U.K. stock returns. The portfolio returns are formed each year and are value-weighted buy-and-hold quarterly returns. I use quarterly data because the consumption data is only available quarterly. All securities are grouped into four portfolios by market value in ascending order (1 to 4). Within each size portfolio, all securities are further grouped into four portfolios by their dividend yield in ascending order (1 to 4). Panels A and B of Table 1 report the mean (Panel A) and the standard deviation (Panel B) of the quarterly excess returns of the 16 size-dividend yield portfolios between 1964 and 2007.

Table 1 here

Panel A of Table 1 shows that there is a size and dividend yield effect in the spread of average excess returns across the 16 portfolios. The small size portfolios (SIZE1DY1 to SIZE1DY4) have higher average excess returns than the corresponding large size portfolios (SIZE4DY1 to SIZE4DY4). Within each size quartile, the high dividend yield portfolio has a higher average excess return than the low dividend yield portfolio. There is a wide cross-sectional spread in the average excess returns across the 16 portfolios ranging between 1.153% (SIZE4DY1) and 3.488% (SIZE1DY2). The data for the primitive assets come from LSPD.

3.2. Linear factor models

The stochastic discount factor for linear factor models is given by:

$$y_t = a + \Sigma^{K}_{k=1} b_k f_{kt} \tag{9}$$

where f_{kt} is the value of factor k at time t, and K is the number of factors. The coefficient a is the constant in the stochastic discount factor, and b_k is the slope coefficient on factor k. I use the following factor models:

1. CAPM

This model is a single-factor model that uses the excess returns of the U.K. stock market index (Market) as the proxy for aggregate wealth. The data for the CAPM model come from LSPD.

2. Fama and French (1993) (FF)

This model model is a three-factor model. The factors are the excess return on the market index and two zero-cost portfolios that capture the size (SMB) and value-growth (HML) effects in stock returns. I use the dividend yield to capture the value-growth effect. The data for the FF model come from LSPD.

3. Jagannathan and Wang (1996) (LCAPM)

The LCAPM model of Jagannathan and Wang (1996) is a two-factor model. The factors are the excess market return and labor income growth (Lab) that proxies for the return on human capital. I use the quarterly per capita growth of the aggregate compensation to employees (e.g., Gao and Huang, 2008) to proxy for labor income growth. The data to form the labor income growth factor come from the Office for National Statistics.

4. ICAPM

This model is a four-factor model based on Campbell (1996) as applied by Petkova (2006). The factors are the excess market return and innovations in the dividend yield of the market index (u_{DY}), the return on the three-month Treasury Bill (u_{Rf}), and the term spread (u_{Term}). The term spread is the difference between the annualized yields on long-term U.K. government bonds and the three-month U.K. Treasury Bill. The long-term bond yield comes from the International Financial Statistics provided by the IMF. The ICAPM specification I use differs from Petkova in that I do not include a default spread due to the unavailability of the default spread data during the whole sample period.

5. Linear CCAPM (CCAPM)

This model is a single linear factor model that uses the log of real aggregate consumption growth (CG) as the factor. I use the quarterly per capita real aggregate consumption growth for non-durables and services. The consumption data come from the Office of National Statistics.

I also consider conditional versions of the CAPM, FF, and CCAPM models where I assume a_{t-1} and b_{kt-1} are a linear function of a single demeaned information variable (z_{t-1}), which is assumed to be in the information set of investors. This approach is similar to Lettau and Ludvigson (2001), Hodrick and Zhang (2001), among others.

3.3. Information variables

The original set of information variables I use in the conditional factor models include the lagged dividend yield on the market index, the lagged quarterly return on the three-month U.K. Treasury Bill, the lagged term spread, and the lagged quarterly growth in the U.K. industrial production index. The data for the industrial production index comes from the International Financial Statistics. Similar instruments have been used in studies by Fama and French (1988), Solnik (1993), Hodrick and Zhang (2001), among others. To examine the predictive ability of the instruments, I run regressions of the quarterly market excess returns on a constant and each information variable in separate regressions. The lagged dividend yield is the only instrument with a significant positive slope coefficient at the 10% significance level and has the highest R^2 of

4.33% across the four instruments. Due to the greater predictive ability of the lagged dividend yield instrument, my tests will use the lagged dividend yield (DY) for the conditional factor models.

4. Empirical results

4.1. All models

I begin by estimating the stochastic discount factor models to minimize the first HJD and second HJD. Table 2 reports summary statistics of the fitted stochastic discount factor values and the first HJD (second HJD) where the model parameters have been estimated to minimize the first HJD (Panel A) or to minimize the second HJD (Panel B). The summary statistics include the mean and standard deviation of the fitted values of the stochastic discount factors. The (Prop < 0) column reports the proportion of the fitted values of the candidate stochastic discount factor that are below zero. The d (d^+) column is the first HJD (second HJD). The *p*-value column is the *p*-value of the null hypothesis that the model has a zero first HJD or second HJD using the tests in Jagannathan and Wang (1996) and Li, Xu, and Zhang (2009). The final column is the standard error of d and d^+ under the null hypothesis that the model is misspecified (e.g., Hansen, Heaton, and Luttmer, 1995). All of the test statistics are corrected for the effects of heteroskedasticity using the method of White (1980).

Table 2 here

Panel A of Table 2 shows that the mean of the fitted stochastic discount factor values for all models is just below one and is similar across models. This result is due to the inclusion of the Treasury Bill return in the set of test assets as recommended by Farnsworth, Ferson, Jackson, and Todd (2002).[4] The volatility of the stochastic discount factor models ranges between 0.166

[4] Kan and Robotti (2008) show that tying down the mean stochastic discount factor value to a sensible value is an important issue in evaluating asset pricing models.

(CAPM) and 1.702 (ICAPM). The conditional version of the CAPM, FF, and CCAPM models has a higher volatility than the unconditional version of the models. The higher volatility leads to a greater proportion of negative stochastic discount factor values. The ICAPM, Cond CAPM, and Cond FF models all have greater than 10% of their fitted stochastic discount factor values below zero.

Panel A of Table 1 also shows that the ICAPM and Cond FF models have the lowest first HJD across the factor models. The first HJD ranges between 0.546 (CCAPM) and 0.350 (ICAPM). The conditional versions of the CAPM, FF, and CCAPM models have a lower first HJD than the corresponding unconditional models. The null hypothesis of a zero first HJD can be rejected for the CAPM, LCAPM, FF, CCAPM, and Cond CCAPM models. For the Cond CAPM, ICAPM, and Cond FF models, the null hypothesis of a zero first HJD cannot be rejected at the 5% significance level. These three models have the most volatile stochastic discount factors and the largest number of negative stochastic discount factor values. These results suggest that the good performance from these models come at the expense of violating the no arbitrage restriction, especially for the ICAPM model. This finding is similar to Wang and Zhang (2006) and Li, Xu, and Zhang (2009). A further consequence of the higher volatility for these three models is the higher standard errors of the first HJD relative to the other models. The larger standard errors make it more difficult to discriminate between the performance of models (e.g., Kan and Robotti, 2009).

Using the second HJD to estimate the model parameters has a dramatic impact on the volatility of the stochastic discount factors and the performance of some models. The mean fitted stochastic factor values are all around the 0.976 and 0.979 region. There is a substantial drop in the volatility of the fitted stochastic discount factor for the ICAPM, Cond CAPM, and Cond FF models. The drop in volatility eliminates most of the negative stochastic discount factor values by the models, especially for the ICAPM model. The Cond FF model has the largest proportion of negative values at 4.55%. The lower volatility also leads to a drop in the standard errors for the second HJD of the ICAPM, Cond CAPM, and Cond FF models. The standard errors are now more similar across models.

Using the second HJD to evaluate the models provides evidence against all the models. The null hypothesis of a zero second HJD is rejected for every model at the 1% significance level. For most models, there is only a small rise in the second HJD relative to the first HJD. However, for the best performing models under the first HJD, the rise is more substantial. The increase in the second HJD relative to the first HJD for the ICAPM model exceeds 40%. This result suggests that the superior performance of the ICAPM model under the first HJD comes at the expense of violating the no arbitrage restriction. The second HJD ranges between 0.397 (Cond FF) and 0.562 (CCAPM). The Cond FF model has the best performance along the two HJDs.

Table 2 suggests that none of the models are able to correctly price the test assets and be arbitrage free at the same time. In the additional tests (not fully reported but available from the author), I examine whether the first HJD and second HJD for two models are equal to each other for every pair of models using the model comparison tests of Kan and Robotti (2009) and Li, Xu, and Zhang (2009). Similar to Kan and Robotti, I find few significant differences in the first HJD between models. In contrast, using the second HJD there are more significant differences in the second HJD between models. The Cond FF model has a significant lower second HJD than all the other models, except the ICAPM model at the 10% significance level which confirms the better performance of the Cond FF model. This result supports the use of conditional factor models as in Hodrick and Zhang (2001), Lettau and Ludvigson (2001), among others.

4.2. Parameter estimates of the best performing models

Table 2 suggests that the no arbitrage restriction has a significant impact on both the estimation and evaluation of linear factor models. In this subsection, I examine the impact of the no arbitrage restriction on the model parameters of the ICAPM, Cond CAPM, and Cond FF models, those which have the lowest first HJD. Table 3 reports the model parameters and *t*-statistics in parentheses for the three models where the model parameters are estimated to minimize the first HJD and second HJD for the ICAPM (Panel A), Cond CAPM (Panel B), and Cond FF (Panel C) models. The Wald test examines the null hypothesis that the slope coefficients on the information variable and scaled factors are jointly equal to zero. The test statistics allow for model misspecification and are based on Kan and Robotti (2009) and Li, Xu, and Zhang (2009).

Table 3 here

Table 3 shows that there are very few significant slope coefficients in the three stochastic discount factor models using the first HJD. There is a significant negative slope coefficient on the Term factor in the ICAPM model. There is a significant negative slope coefficient on the Market factor in the Cond CAPM model. The Wald test is unable to reject the null hypothesis that the slope coefficients on the information variable and scaled factors are jointly equal to zero for both conditional models. The lack of statistical significance of the stochastic discount factor coefficients and the Wald test using the first HJD stems from controlling for possible model misspecification as in Kan and Robotti (2009).

The no arbitrage restriction has two effects on the estimation of the stochastic discount factor coefficients in Table 3. The first is that the use of the second HJD to estimate the model parameters moderates the size of the coefficients for the majority of factors. All of the slope coefficients in the ICAPM and Cond CAPM models are smaller in absolute terms using the second HJD. Four out of the eight slope coefficients are smaller in absolute terms in the Cond FF model. The smaller slope coefficients explain why the volatility of the fitted stochastic discount factor values is lower for the three models when estimating the model parameters to minimize the second HJD. This pattern in coefficients is similar to Bailey, Li, and Zhang (2004) and Wang and Zhang (2006).

The second impact of the no arbitrage restriction is that the statistical significance of the stochastic discount factor coefficients increases. This result stems in most cases from lower standard errors than higher slope coefficients. In the ICAPM model, there is a significant negative coefficient on the Market factor. In the Cond CAPM model, there is a significant positive coefficient on the lagged dividend yield instrument and a significant negative slope coefficient on the Market factor. In the Cond FF model, there is a significant positive slope coefficient on the lagged dividend yield instrument and a significant negative slope coefficient on the Market and HML factors. For both models, the Wald test can reject the null hypothesis that the slope coefficients on the information variable and scaled factors are jointly equal to zero.

The significant Wald tests using the second HJD provide further support for the usefulness of conditional factor models in helping to price the test assets. The use of the second HJD might also mitigate some of the concerns raised by Lewellen and Nagel (2006) about the superior performance of the conditional factor models. Lewellen and Nagel argue that the slope coefficients on the scaled factors are too large to be consistent with the theory of conditional models. Using the second HJD to estimate the conditional models leads to smaller slope coefficients on the scaled factors, excluding the scaled market factor in the Cond FF model, compared to using the first HJD so it should mitigate some of these concerns.[5]

4.3. Model pricing errors

This subsection examines the impact of the no arbitrage restriction on the pricing errors of the ICAPM, Cond CAPM, and Cond FF models. I also include the CAPM and FF models for comparison. I estimate the pricing errors of the 16 size-dividend yield portfolios for the five models using both the first HJD and the second HJD. Figures 1 and 2 plot the pricing errors across the 16 size-dividend yield portfolios using the first HJD (Figure 1) and the second HJD (Figure 2). To conserve space, I do not include the corresponding *t*-statistics.

Figure 1 here

Figure 2 here

Figure 1 shows the poor performance of the CAPM in pricing the size-dividend yield portfolios using the first HJD. The poor performance is concentrated in the bottom two size quartiles where there are large pricing errors. All of the portfolios in the smallest two size quartiles, except SIZE2DY1, have significant positive pricing errors. The two most challenging portfolios are the SIZE1DY2 and SIZE1DY3 portfolios where the pricing errors exceed 2%. There is a clear size effect in the pricing errors for the CAPM, where the pricing errors for the bottom size quartile are larger than the top size quartile. The dividend yield effect in the pricing errors is most noticeable in the top size quartile and to a lesser extent in the next largest size quartile. The

[5] In unreported tests (available from the author), I estimate the time varying factor risk premiums as in Lettau and Ludvigson (2001) for the factors in the Cond CAPM and Cond FF models using the first HJD and second HJD. I find that the conditional factor risk premiums are less volatile when using the second HJD for all factors except the conditional market risk premium in the Cond FF model.

CAPM has the highest Root Mean Squared Pricing Error (RMSE) of 1.12% across all five models.

Figure 1 shows that the FF, ICAPM, Cond CAPM, and Cond FF models all make substantial improvements in reducing the pricing errors of the CAPM using the first HJD. The RMSE for these models is 0.54% (FF), 0.37% (ICAPM), 0.63% (Cond CAPM), and 0.33% (Cond FF). The best performing models by the RMSE metric are the ICAPM and Cond FF models. Both models reduce the pricing errors of the SIZE1DY2 and SIZE1DY3 portfolios to be under 0.7%. There is also less of a size effect and dividend yield effect in the pricing errors for the ICAPM and Cond FF models.

Figure 2 shows that the no arbitrage restriction has little impact on the pricing errors for the CAPM model. The SIZE1DY2 and SIZE1DY3 portfolios continue to have the largest pricing errors, which exceed 2%. The RMSE for the CAPM using the second HJD is 1.13%, the highest across the five models. The RMSE for the other four models is 0.56% (FF), 0.81% (ICAPM), 0.81% (Cond CAPM), and 0.37% (Cond FF). The use of the second HJD has a marginal impact on the pricing errors for the FF and Cond FF models. However, the no arbitrage restriction has a dramatic impact on the pricing errors for the ICAPM and Cond CAPM models.

The no arbitrage restriction has two effects on the pricing errors of the ICAPM and Cond CAPM models. The first effect is a sharp rise in the magnitude of the pricing errors. For the ICAPM model, the RMSE using the second HJD is more than double the RMSE using the first HJD. There is a clear dividend yield effect in the pricing errors of the four portfolios in the largest size quartile. The pricing errors of the four portfolios in the smallest size quartile are likewise larger than the corresponding pricing errors of the four portfolios in the largest size quartile. A similar pattern emerges for the Cond CAPM model but to a lesser extent than the ICAPM model.

The second effect of the no arbitrage restriction is to increase the statistical significance of the pricing errors for the ICAPM and Cond CAPM models. Using the first HJD, not one of the pricing errors is significant at the 5% level for the ICAPM and Cond CAPM models. Using the second HJD, there are five significant positive pricing errors for the ICAPM model and four significant positive pricing errors for the Cond CAPM model. The SIZE1DY2 and SIZE1DY3 portfolios have the largest significant positive pricing errors for both models, which exceed 1.5%.

Figure 2 shows that the Cond FF model has the best performance using the second HJD. Although the Cond FF model has some significant pricing errors, there is a substantial reduction in the magnitude of the pricing errors, especially for the SIZE1DY2 and SIZE1DY3 portfolios. The size and dividend yield patterns in the pricing errors is also less evident for the Cond FF model.

The findings in Tables 2 and 3 and Figures 1 and 2 highlight the superior performance of the Cond FF model. In the additional tests (not fully reported but available from the author), I examine the out-of-sample pricing performance of the five models using the excess returns of the ten momentum portfolios[6] and the ten cluster portfolios (e.g., Ahn, Conrad, and Dittmar, 2009). I estimate the pricing errors of the ten momentum portfolios and ten cluster portfolios using the estimated stochastic discount factor for each model using the first HJD and second HJD from the primitive assets. I find that the Cond FF model has poor out-of-sample pricing performance in both the momentum and cluster portfolios. The Cond FF model has the highest RMSE across the five models using the momentum portfolios. I also find that the no arbitrage restriction has a limited impact in improving the out-of-sample pricing performance of the models. There is a marginal reduction in the RMSE for the momentum portfolios across all models when using the es-

[6] The momentum portfolios are formed each year where the securities are ranked on the basis of their past average returns during months -12 to -2.

timated stochastic discount factor from the second HJD. The results are more mixed when using the cluster portfolios.

5. Conclusions

This paper examines the impact of the no arbitrage restriction on the estimation and evaluation of linear factor models in U.K. stock returns. There are three main findings in my study. First, the no arbitrage restriction has a significant impact on the evaluation of linear factor models. All of the models are rejected using the second HJD. The second HJD is more able to detect significant differences in pricing performance between models compared to the first HJD. This finding is similar to Li, Xu, and Zhang (2009).

Second, the use of the no arbitrage restriction has a significant impact on the estimation of linear factor models. Using the second HJD to estimate the models reduces the volatility of the fitted stochastic discount factor values and eliminates most of the negative values. The absolute size of the coefficients in the stochastic discount factor models are reduced for most factors and scaled factors. These results are similar to Bailey, Li, and Zhang (2004), Wang and Zhang (2006), and Li, Xu, and Zhang (2009). The reduction in the size of the coefficients on the scaled factors moderates some of the concerns raised in Lewellen and Nagel (2006) about the superior performance of conditional models.

Third, the no arbitrage restriction has a significant impact on the magnitude and statistical significance of the pricing errors for some models. For the ICAPM model which performs well using the first HJD, there is a large increase in the magnitude and statistical significance of the pricing errors using the second HJD. The poor performance of the ICAPM model using the second HJD suggests that the model performs well using the first HJD at the expense of violating the no arbitrage requirement and supports the results in Wang and Zhang (2006) and Li, Xu, and Zhang (2009).

The best performing model I consider is the Cond FF model. It has the lowest second HJD and significantly outperforms a number of the other models. However the Cond FF model performs poorly in out-of-sample pricing performance tests. I also find that the use of the second HJD to estimate models has only a limited impact in improving the out-of-sample pricing performance of linear factor models. These results suggest we still haven't found a good model that can capture the risk and return dynamics in U.K. stock returns. A number of questions could be explored in future research. One issue worth examining would be to use the second HJD to evaluate nonlinear models based on the three-moment and four-moment CAPM (e.g., Dittmar 2002). A fuller examination of whether the second HJD can improve the reliability of linear factor models in practical applications such as fund performance or cost of capital estimation is warranted. I leave these issues to future research.

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Table 1
Summary statistics of primitive assets and factors

The table reports summary statistics of the quarterly excess returns (%) of 16 size-dividend yield (DY) portfolios (Panels A and B) between Q1 1964 and Q4 2007. The summary statistics include the mean and standard deviation. The portfolios are numbered ij, where i refers to size rising from one to four and j refers to the dividend yield rising from one to four.

Panel A: Means

Portfolios	DY1	DY2	DY3	DY4
SIZE1	2.615	3.488	3.351	2.960
SIZE2	2.487	2.380	2.961	2.768
SIZE3	1.605	1.795	2.401	2.445
SIZE4	1.153	1.381	1.808	2.408

Panel B: Standard deviations

Portfolios	DY1	DY2	DY3	DY4
SIZE1	11.290	10.182	9.999	11.403
SIZE2	11.454	10.439	11.086	11.531
SIZE3	11.735	10.990	11.665	12.214
SIZE4	11.459	10.524	10.309	10.414

Table 2
The performance of candidate models

The table reports the Hansen and Jagannathan (1997) distance measures and summary statistics of the fitted values of the candidate stochastic discount factor models between Q1 1964 and Q4 2007. In Panel A (Panel B) of the table, the model parameters are estimated to minimize the first (second) HJD. The summary statistics are the mean and standard deviation. Prop < 0 is the proportion (%) of the fitted values of the stochastic discount factor that are negative. The d (d^+) column is the first (second) HJD. The *p*-value (d (d^+)) column is the *p*-value that tests the null hypothesis of a zero first (second) HJD. The final column is the standard error (SE) of the first (second) HJD under the null of a misspecified model. The test statistics are corrected for the effects of heteroskedasticity using the method of White (1980).

Panel A: First HJD

	Mean	Standard deviation	Prop < 0 (%)	d	*p*-value (d=0)	SE(d)
CAPM	0.979	0.166	0.568	0.531	0.000	0.075
LCAPM	0.978	0.292	0.000	0.523	0.000	0.076
FF	0.979	0.288	0.568	0.479	0.000	0.077
ICAPM	0.981	1.702	26.136	0.350	0.863	0.145
CCAPM	0.979	0.309	0.000	0.546	0.000	0.076
Cond CAPM	0.975	1.013	14.204	0.439	0.149	0.102
Cond FF	0.976	0.979	10.227	0.358	0.164	0.100
Cond CCAPM	0.978	0.737	2.841	0.513	0.013	0.083

Panel B: Second HJD

	Mean	Standard deviation	Prop < 0 (%)	d^+	*p*-value (d^+=0)	SE(d^+)
CAPM	0.979	0.165	0.568	0.544	0.000	0.082
LCAPM	0.978	0.277	0.000	0.536	0.000	0.083
FF	0.979	0.288	0.568	0.493	0.000	0.084
ICAPM	0.979	0.515	2.841	0.500	0.000	0.095
CCAPM	0.979	0.195	0.000	0.562	0.000	0.084
Cond CAPM	0.977	0.599	3.977	0.476	0.003	0.093
Cond FF	0.977	0.641	4.546	0.397	0.009	0.093
Cond CCAPM	0.978	0.543	0.568	0.529	0.001	0.088

Table 3
Parameter estimates under the first and second HJD

The table reports the stochastic discount factor coefficients and *t*-statistics in parentheses for the ICAPM, Cond CAPM, and Cond FF models. The model parameters are estimated to either minimize the first or second HJD between Q1 1964 and Q4 2007. Market is the excess returns on the value weighted market index. SMB and HML are zero-cost portfolios of the size and value-growth effects in U.K. stock returns. The u_{DY}, u_{Rf}, and u_{Term} are the residuals on the dividend yield on the market index, the return on the three-month U.K. Treasury Bill, and the term spread from a first-order VAR. DY is the lagged dividend yield on the market index. The Wald test examines the null hypothesis that the coefficients of the information variable and scaled factors are jointly equal to zero for the two conditional models. The test statistics allow for model misspecification and are corrected for the effects of heteroskedasticity using the method of White (1980).

Panel A: ICAPM

	Const	Market	u_{DY}	u_{Rf}	u_{Term}
First HJD	1.007	-1.688	5.659	-17.025	-20.938
	(7.405)**	(-1.113)	(0.979)	(-1.854)	(-2.055)*
Second HJD	1.008	-1.632	3.533	-3.926	-5.296
	(13.081)**	(-2.264)*	(0.297)	(-0.271)	(-0.465)

Panel B: Cond CAPM

	Const	DY	Market	Market*DY
First HJD	1.023	80.223	-4.630	107.655
	(11.502)**	(1.629)	(-2.424)*	(0.751)
Wald	3.645			
Second HJD	1.014	45.674	-3.466	73.905
	(14.242)**	(3.812)**	(-2.599)**	(1.000)
Wald	21.464**			

Panel C: Cond FF

	Const	DY	Market	SMB	HML	Market *DY	SMB *DY	HML *DY
First HJD	0.988	59.672	-3.583	0.583	-6.522	133.892	-554.743	-702.824
	(9.625)**	(0.816)	(-1.928)	(0.254)	(-1.350)	(0.562)	(-0.674)	(-1.177)
Wald	7.061							
Second HJD	1.027	34.835	-3.712	0.264	-7.158	184.630	-128.460	-345.250
	(18.391)**	(2.049)*	(-2.115)*	(0.180)	(-2.364)*	(1.579)	(-0.291)	(-1.145)
Wald	36.335**							

** and * indicate statistical significance at the 1% and 5% levels, respectively.

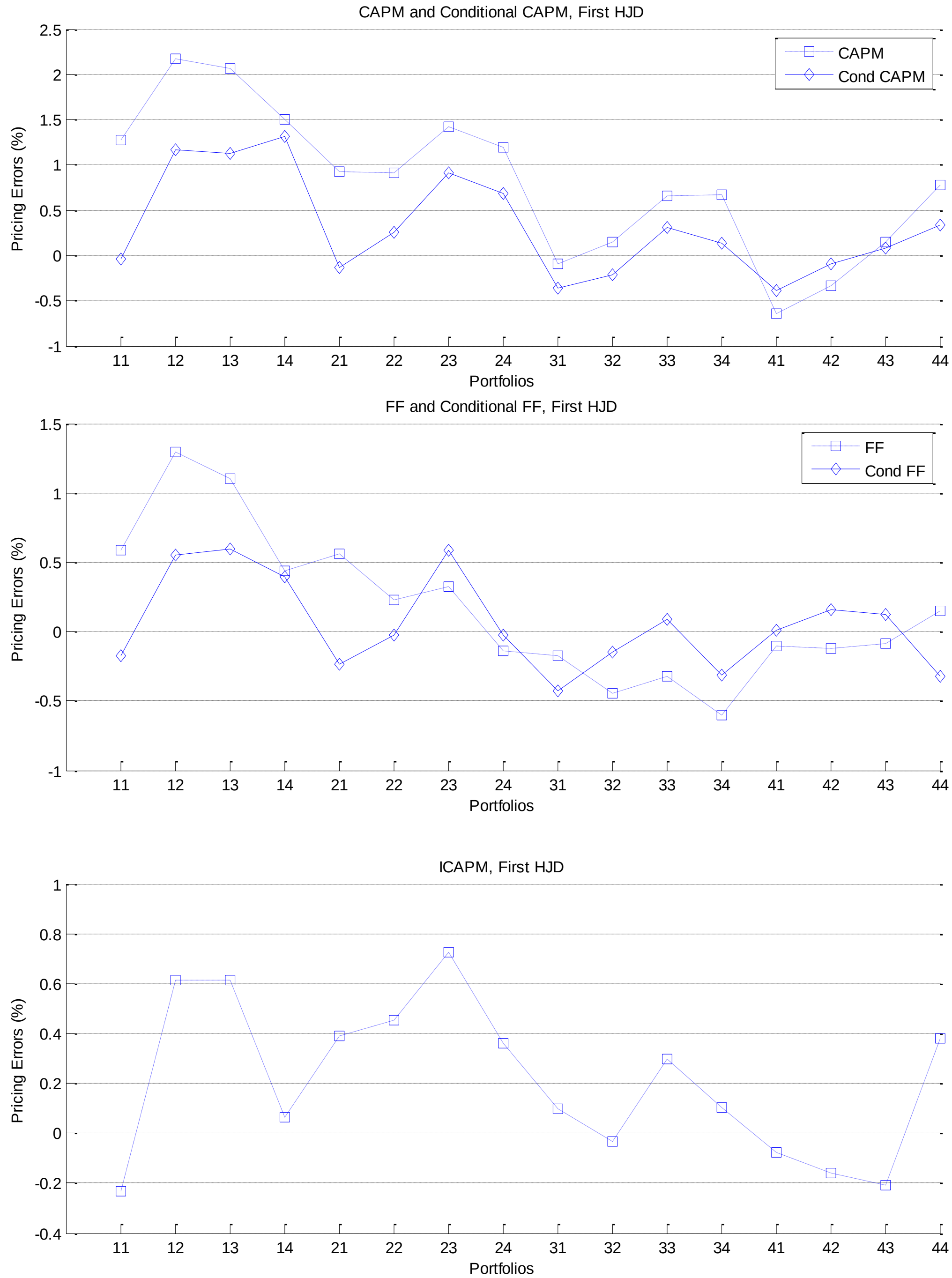

Figure 1 **Model pricing errors using the first HJD**

The graph depicts the pricing errors of the 16 size-dividend yield portfolios for the CAPM, FF, ICAPM, Cond CAPM, and Cond FF models using the first HJD. The sample period covers the Q1 1964 and Q4 2007 period. The primitive assets are the quarterly excess returns of the 16 size-dividend yield portfolios and the quarterly gross return of the three-month U.K. Treasury Bill. The first number in the portfolio name is the size quartile it belongs to and the second number is the dividend yield quartile it belongs to. The size and dividend yield groups are in ascending order.

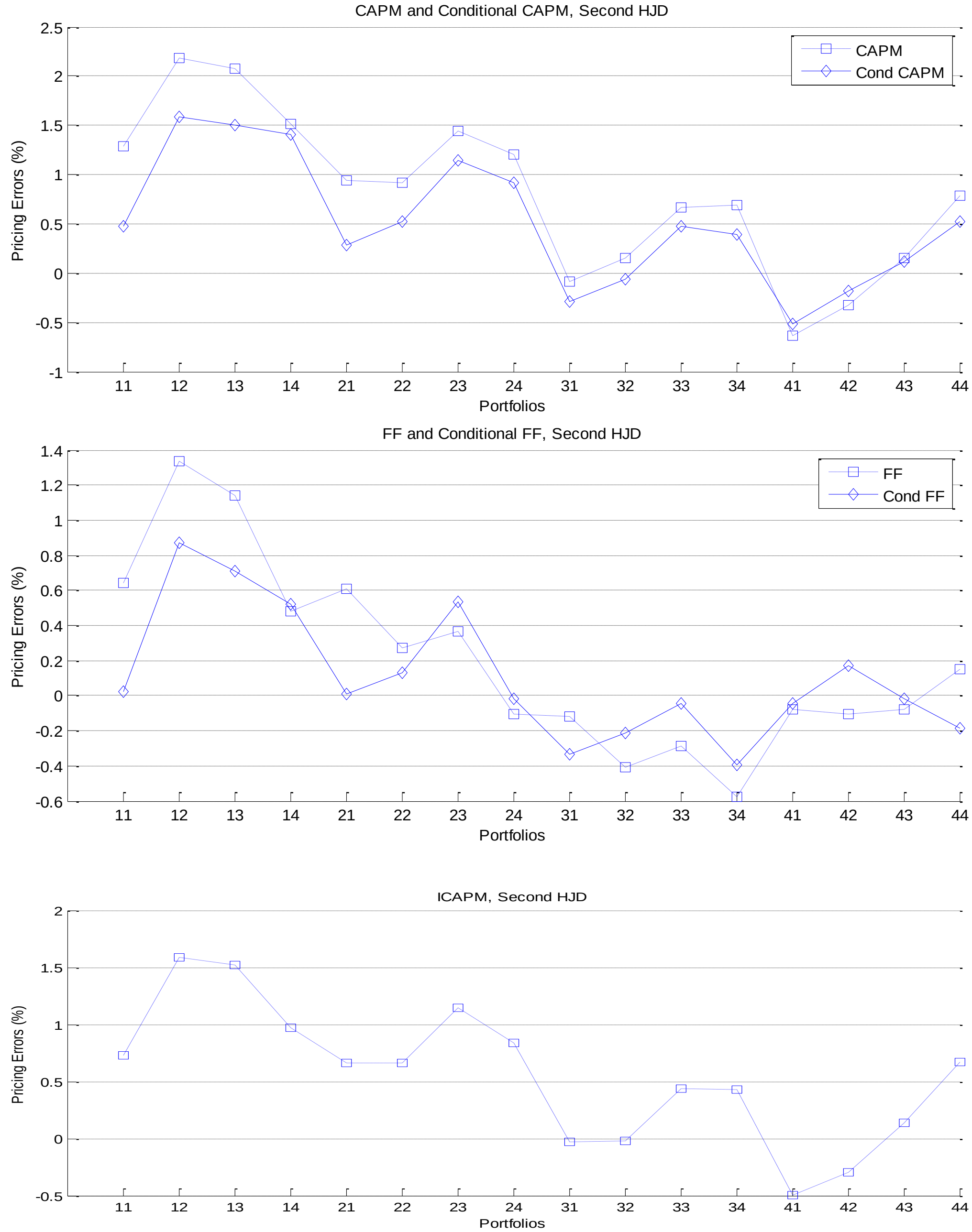

Figure 2 **Model pricing errors using the second HJD**

The graph depicts the pricing errors of the 16 size-dividend yield portfolios for the CAPM, FF, ICAPM, Cond CAPM, and Cond FF models using the second HJD. The sample period covers the Q1 1964 and Q4 2007 period. The primitive assets are the quarterly excess returns of the 16 size-dividend yield portfolios and the quarterly gross return of the three-month U.K. Treasury Bill. The first number in the portfolio name is the size quartile it belongs to and the second number is the dividend yield quartile it belongs to. The size and dividend yield groups are in ascending order.