

Extreme correlation of stock and bond futures markets:

International evidence

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Abstract

This study explores time-varying extreme correlation of stock-bond futures markets in three major developed countries. In the U.S. and the UK, there is evidence of positive extreme stock-bond correlation when both futures markets are extremely bullish or bearish. In Germany, stock-bond futures extreme correlation is negative, suggesting the most diversification potentials of bond futures when German stock index futures market plunges. Macroeconomic news, the business cycle, and the stock market uncertainty all significantly affect the median stock-bond futures correlation. However, only the stock market uncertainty still significantly affects the extreme stock-bond futures correlation when the stock market is extremely bearish.

JEL Classification: G12, G15, E44

Key Words: Stock-bond extreme correlation; futures; copula; macroeconomic news; stock market uncertainty

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1. Introduction

Stocks and bonds are the two most important asset classes, which play a crucial role in asset allocation and portfolio management. It is now the stylized fact that the (conditional) correlation between stock and bond returns has varied considerably in the United States and other developed countries (e.g., Baele, Bekaert, and Inghelbrecht, 2010; Scruggs and Glabadanidis, 2003). Efforts also have been made to explore various economic forces driving the time-varying stock-bond correlation, including stock market uncertainty (Connolly, Stivers, and Sun, 2005, 2007), macroeconomic news (Brenner, Pasquariello, and Subrahmanyam, 2009), and business cycle indicators (Jensen and Mercer, 2003; Andersen, Bollerslev, Diebold, and Vega, 2007; Yang, Zhou, and Wang, 2009).

Our study employs daily stock and bond futures data for three major developed markets (the U.S., the UK, and Germany) to examine time-varying stock-bond extreme correlation.¹ We contribute to existing research in the following aspects.

¹ While many earlier studies (e.g., Scruggs and Glabadanidis, 2003; Yang, Zhou, and Wang, 2009, 2010; Baele, Bekaert and Inghelbrecht, 2010) focus on monthly (or even lower quarterly) frequency, much of the recent literature (e.g., Andersen, Bollerslev, Diebold, and Vega, 2007; Brenner, Pasquariello, and Subrahmanyam, 2009; Connolly, Stivers, and Sun, 2005, 2007; Kim, Moshirian, and Wu, 2006) explores stock-bond return relation at daily (or higher) frequency, which is followed in this study. According to Kim, Moshirian, and Wu (2006, p.1510), it is important to focus on daily frequency because comovements in the stock and bond returns often change on a rapid basis as investors shift their domestic asset allocation. Connolly, Stivers, and Sun (2005, p.189) also argue that the magnitude

First, unlike many previous studies (e.g., Gulko, 2002; Scruggs and Glabadanidis, 2003; Hartmann, Straetmans, and de Vries, 2004; Connolly, Stivers, and Sun, 2005; Kim, Moshirian, and Wu, 2006; Yang, Zhou, and Wang, 2009), we focus on stock and bond futures market returns. With a notable exception of Andersen, Bollerslev, Diebold, and Vega (2007), our study is among the first few to investigate international stock and bond futures markets. The use of futures market data avoids the notorious nonsynchronous trading problem for daily stock index data, as documented in Lo and Mackinlay (1988) and reemphasized in Ahn, Boudoukh, Richardson, and Whitelaw (2002), the latter of which illustrates the different time series properties between stock index cash and futures returns. Furthermore, stock-bond relations, as reflected in the futures markets, arguably, may be more relevant to active traders. Replicating the stock market index by buying constituent stocks involves a greater initial investment, higher transaction costs, and more tracking errors problems. By contrast, stock index futures trading is preferred by investors, particularly those engaged in speculative transactions (Kawaller, Koch, and Koch, 1987).

In addition, unlike many existing studies, we are among the first to explore conditional extreme correlation rather than conditional (median) correlation between stock and bond markets. Such a consideration would be crucial to exploring diversification benefits of bond futures markets to stock futures market investors when it is most needed (i.e., when the stock futures market crashes).² The

and reliability of the time variations of stock-bond comovement likely would be less for longer horizon (e.g., monthly) returns since fewer observations are available for measuring time-varying correlations.

² As discussed in Connolly, Stivers, and Sun (2005, 2007) and Kim, Moshirian, and Wu (2006), daily stock-bond return relation carries the implication for a timely diversification benefit for domestic asset

copula-based GARCH model we use extends earlier studies on stock-bond interactions using GARCH models (e.g., Scruggs and Glabadanidis, 2003; Kim, Moshirian, and Wu, 2006; Brenner, Pasquariello, and Subrahmanyam, 2009). More importantly, extending numerous copula-based studies in the literature (e.g., Kole, Koedijk, and Verbeek, 2007; Garcia and Tsafack, 2011), we propose to use the diagonal (i.e., lower-upper or upper-lower) tail dependence as the new measure of diversification potential, rather than widely used lower-lower or upper-upper tail dependence that is suited for examining contagion across markets.³ Such a measure can shed new light on many important portfolio and risk management issues, such as searching for safe havens for investors during financial turmoil as discussed in Baur and Lucey (2010) and Baur and McDermott (2010).

Finally, we investigate how various economic factors affect the stock-bond extreme correlation, and further contrast with their effects on the stock-bond correlation. Baele, Bekaert, and Inghelbrecht (2010) reveal that while it is important to disentangle economic sources of time variation in stock-bond co-movement, not much work has been done. In this regard, understanding economic sources of extreme correlation has not yet been addressed in the research, which is a gap our study attempts to fill. Also, the types of economic factors under consideration are quite

allocation, which stands in contrast to the well-explored cross-equity market international diversification.

³ While they do not use copula-based modeling, Hartmann, Straetmans, and de Vries (2004) use an alternative extremal dependence measure to investigate upper-lower and lower-upper tail dependence between stock and bond markets in G-5 countries. Our study differs from theirs by focusing on daily frequency, futures markets, time variations, and the potential determinants of such stock-bond correlation. While Gulko (2002) also makes a similar point, he only focuses on a few event windows of stock market crashes and does not conduct further analysis of determinants of extreme stock-bond correlation. Finally, using higher moment tail risk measures to shed light on the diversification benefit (e.g., You and Daigler, 2010) can be an alternative to the copula-based framework used in our study.

comprehensive, as they include major macroeconomic news announcements (e.g., Brenner, Pasquariello, and Subrahmanyam, 2009), stock market volatility (Connolly, Stivers, and Sun, 2005, 2007), and business cycle indicators (dummy variables for recession/contraction versus expansion) (e.g., Andersen, Bollerslev, Diebold, and Vega, 2007).

2. Methodology

2.1. Diagonal tail dependence coefficients

Copula has been used extensively to model the dependence structure of a joint distribution (Sklar, 1959; Nelsen, 1999; Joe, 1997). Both upper-upper tail and lower-lower tail dependence capture that dependence when both assets move in the same direction at the extreme (either up or down simultaneously). However, in many circumstances, financial assets may move in the *opposite* direction at the extreme. Both upper-upper tail and lower-lower tail dependence measures are inadequate to capture these kinds of extreme dependence. Motivated by this fact, we propose an alternative measure for the tail dependence at the lower-upper and upper-lower tail. To the best of our knowledge, the concept of copula is primarily applied to estimate the lower-lower tail and the upper-upper tail dependence coefficients, and no existing studies apply the cross-tail dependence coefficients in financial research.

Analogous to the upper-upper and lower-lower tail-dependence, the upper-lower and lower-upper tail dependence coefficients, by definition, are as follows:

$$\lambda_{UL} = \lim_{u \rightarrow 0^+} \Pr[F_Y(Y) \leq u \mid F_X(X) > 1 - u] = \lim_{u \rightarrow 0^+} \frac{u - C(1 - u, u)}{u} \quad (1)$$

$$\lambda_{LU} = \lim_{u \rightarrow 1^-} \Pr[F_Y(Y) > 1 - u \mid F_X(X) \leq u] = \lim_{u \rightarrow 1^-} \frac{1 - u - C(1 - u, u)}{1 - u} \quad (2)$$

The lower-upper (upper-lower) tail dependence coefficient quantifies the probability to observe a large (small) Y , given that X is small (large).

We can use the definition of lower-lower and upper-upper tail dependence coefficients to *derive* lower-upper and upper-lower tail dependence coefficients. Let $U = F_X(X)$ and $V = F_Y(Y)$ where U and V are distributed uniformly on $[0, 1]$. Using the transformation $(U, V) = (U, 1 - V)$ and the definition of the lower-lower tail dependence coefficient, we obtain:

$$\begin{aligned} \lambda_{LL} &= \lim_{u \rightarrow 0^+} \Pr(V \leq u \mid U \leq u) \\ &= \lim_{u \rightarrow 0^+} \Pr(1 - V' \leq u \mid U' \leq u) \\ &= \lim_{u \rightarrow 0^+} \Pr(V' > 1 - u \mid U' \leq u) \\ &= \lambda_{LU} \end{aligned}$$

In other words, if we keep the random variable U to be the same and transform the random variable V into $1 - V$, and apply the definition of the lower-lower tail dependence coefficient, we will automatically obtain the lower-upper tail dependence coefficient.⁴ The transformation $(U, V) = (U, 1 - V)$ rotates the $U - V$ panel 90 degrees clockwise.

Using the same transformation, the upper-lower tail dependence coefficient can be obtained from the upper-upper tail dependence coefficient, since

⁴ We thank Harry Joe at the University of British Columbia for pointing out this transformation.

$$\begin{aligned}
\lambda_{UU} &= \lim_{u \rightarrow 1^-} \Pr(V > u \mid U > u) \\
&= \lim_{u \rightarrow 1^-} \Pr(1 - V' > u \mid U' > u) \\
&= \lim_{u \rightarrow 0^+} \Pr(1 - V' > 1 - u \mid U' > 1 - u) \\
&= \lim_{u \rightarrow 0^+} \Pr(V' \leq u \mid U' > 1 - u) \\
&= \lambda_{UL}
\end{aligned}$$

2.2. Choices of the marginal densities and the copula function

We adopt a two-step approach. Following Joe and Xu (1996) and Patton (2006a, 2006b), we fit each return series by the AR-GARCH model. Based on the fitted models, we calculate the implied standardized residuals from the AR-GARCH model. The standardized residuals obtained in the first step estimation will be the input for the copula density estimation in the second step. We use the empirical cumulative distribution function to estimate the marginal densities of the standardized residuals to avoid any misspecification from the parametric assumption. It can be shown that as the sample size goes to infinity, the empirical cumulative density function (cdf) converges uniformly to the true cdf of the standardized residuals.

The next step is to decide the parametric functional form of the copula. There are a variety of copulas in the literature. Each copula captures a different pattern of the dependence structure on the data. The strength of the dependence is determined by the parameters in the parametric copula. Basically, we are looking for the copula that exhibits asymmetric tail dependence feature. A popular choice is the “symmetrized Joe-Clayton” (SJC) copula, with or without time variation. The original Joe-Clayton (JC) copula is defined as:

$$C_{JC}(u, v \mid \lambda^{UU}, \lambda^{LL}) = 1 - (1 - \{ [1 - (1 - u)^\kappa]^{-\gamma} + [1 - (1 - v)^\kappa]^{-\gamma} - 1 \}^{-\frac{1}{\gamma}})^{\frac{1}{\kappa}}$$

$$\begin{aligned} \text{where} \quad \kappa &= 1 / \log_2(2 - \lambda^{UU}) \\ \gamma &= -1 / \log_2(\lambda^{LL}) \\ \text{and} \quad \lambda^{UU} &\in (0, 1), \quad \lambda^{LL} \in (0, 1) \end{aligned}$$

The SJC copula has the expression

$$C_{SJC}(u, v | \lambda^{UU}, \lambda^{LL}) = 0.5 \cdot (C_{JC}(u, v | \lambda^{UU}, \lambda^{LL}) + C_{JC}(1-u, 1-v | \lambda^{UU}, \lambda^{LL}) + u + v + 1)$$

which is based on the modification of the JC copula and addresses the asymmetry problem even though the upper-upper and lower-lower tail dependence measures are equal in the JC copula.

As a benchmark comparison, the Normal copula is the second candidate considered in our study. We also compare these two copulas with other commonly used copulas to assess their relative performance. Three selection criteria are computed: Akaike information criteria (AIC), Bayesian information criteria (BIC), and the value of log-likelihood in the estimated model. In addition, we employ the chi-square ‘‘hit test,’’ suggested by Patton (2006a) to discriminate between different copula models.

We follow Patton (2006a) to allow for a time variation in the dependence coefficients of the asset returns in both SJC and Normal copulas. The following ARMA process for the dependence parameters will be used throughout our study. For symmetrized Joe-Clayton (SJC) copula:

$$\lambda_t^i = \Lambda \left(\beta_0 + \beta_1 \lambda_{t-1}^i + \alpha_i \sum_{j=1}^p |u_{t-j} - v_{t-j}| \right)$$

where $i = UU, LL, UL$ and LU and $\Lambda(x) \equiv (1 + e^{-x})^{-1}$ is the logistic transformation to keep the tail dependence coefficients λ_t^i to lie in the range of $(0, 1)$. For the Normal copula:

$$\rho_t = \tilde{\Lambda}(\eta_0 + \eta_1 \rho_{t-1} + \alpha \sum_{j=1}^p \Phi^{-1}(u_{t-j}) \Phi^{-1}(v_{t-j}))$$

where $\tilde{\Lambda}(x) \equiv (1 - e^{-x}) / (1 + e^{-x}) = \tanh(x/2)$ is the modified logistic transformation to keep ρ_t in the range of $(-1, 1)$. The purpose of the lag of the dependence parameter on the right hand side is to capture the persistence in the dependence parameters.

3. Empirical results

3.1. Data

To examine the extreme correlation between stock and bond futures returns, we use the data of daily stock index futures prices of the U.S. (Standard & Poor's 500 stock index), UK (FTSE 100 index) and Germany (DAX 30 index), and the government bond futures prices for the three countries.⁵ Specifically, following the convention, daily continuous nearby futures price series are constructed by using daily settlement prices for the nearby futures contract at a particular point in time until the contract reaches the first day of the delivery month. Then, prices for the next nearby contract are used. The nearby futures contract is used because it is highly liquid and the most active. The sample periods covered are from May 1984 to December 2008

⁵ Due to the main interest of our study and particularly computational difficulty in estimating a higher-dimensional copular model, we do not consider jointly estimating stock and bond futures returns across the three countries, which is similar to earlier studies (e.g., Connolly, Stivers, and Sun, 2007; Kim, Moshirian, and Wu, 2006).

for the U.S., from January 1983 to December 2008 for the UK, and from October 1998 to December 2008 for Germany. All futures returns series are calculated from the log-difference of the futures prices.

Table 1 gives summary statistics for the data. Standard deviations reveal that the volatilities of index returns are higher than those of the bond returns for the three countries considered in our sample. The most volatile market is Germany, followed by the U.S. and the UK in terms of stock futures returns, while the U.S. exhibits the highest volatility in terms of bond futures returns. All in the series exhibit excessive skewness and kurtosis relative to the Gaussian distributions. Positive kurtosis is more severe in the stock index futures returns than the bond futures returns. The quantile also reveals that marginal distributions of stock returns tend to be more spread out than bond returns. With very small p -values, the Jarque-Bera test also demonstrates the non-normality of each series, which implies the violation of multivariate Gaussian distribution assumption.

To investigate the co-movement of stock-bond futures returns within the countries, we first calculate the linear correlation and Kendall's τ rank correlation between stock and bond returns for the three countries. The patterns revealed by two dependence measures are qualitatively similar. The linear correlations range from -0.35 (Germany) to 0.09 (U.S.), while the Kendall's τ range from -0.21 (Germany) to 0.09 (UK). Germany is the only country that shows an inverse relation between stock and bond returns.

3.2. Asymmetric Dependence Detection

Before proceeding to the statistical modeling of the asymmetric dependence, we need to examine whether the asymmetric dependences are statistically significant among the countries considered. Many recent studies examine the presence of asymmetric dependence between different assets. For example, Login and Solnik (2001), Ang and Chen (2002), and Patton (2006a) use the measure of exceedance correlation, which is defined as the correlation between two asset returns x and y , conditional on both variables being above or below certain thresholds θ_1 and θ_2 .

The graphical results (available upon request) reveal that asymmetric patterns exist in our sample with the correlations at lower quantile values being higher than upper quantile values. Nevertheless, a weakness of the exceedance correlation is that it is computed only from those observations that are below or above the threshold, which means that, as we move further into the tails, the exceedance correlation may be measured less precisely due to the use of fewer and fewer data points.

Thus, we further investigate the extent of asymmetric dependence between stock and bond futures returns of the three countries by conducting a formal statistical test based on the J statistic (Hong, Tu, and Zhou, 2007). The J statistic is a kernel-based model free statistic that tests asymmetric dependence non-parametrically by comparing conditional correlation across opposing regions within a given set of data. When dependence is symmetric across upper and lower tails, the J statistic is close to zero. Otherwise, the presence of asymmetric dependence between upper and lower tails would result in a significant, nonzero J statistic.

We apply the test of correlation symmetry to the three countries. The results for

the test of asymmetric dependence based on the J statistic⁶ are not reported but available on request. All the p -values from the test are less than 0.05 with the U.S. showing the strongest statistically significant correlation asymmetry. Therefore, we reject the null hypothesis that the dependence is symmetric across upper and lower tails, which suggests that the dependence between the stock market and bond futures market is different at different market conditions.

3.3. Goodness of fit test and comparison

By the Sklar theorem (Sklar, 1959), any joint density can be decomposed into its marginal density and a copula function. Common practice adopts a two-step estimation strategy to recover the joint density function; in the first stage, estimate the marginal density of individual data series and then fit the copula density in the second stage. Along this line, we fit each pair of stock-bond return data using a two-step copula-GARCH model. We fit each return series (marginal density) by a AR(1)-GARCH(1,1)⁷ model using the maximum likelihood estimation. Based on the fitted model, we calculate the implied standardized residuals. The cumulative distribution functions of the standardized residuals were estimated by the empirical cumulative distribution method. The empirical cdf obtained in the first step estimation will be the input for the copula density estimation in the second step.

⁶ Quantiles are used to determine cutoff exceedances. This is performed for the cutoff quantile={0.5, 0.6, 0.7, 0.8, 0.9} in conjunction with the Bartlett kernel (Hong, Tu and Zhou, 2007) and the lag truncation order procedure (Newey and West, 1994) for the estimation of the variance-covariance matrix, $\hat{\Omega}$. The p -value of the J -statistic is the calculated from asymptotic chi-square distribution χ_5^2 .

⁷ As suggested by a referee, we conducted the robustness check by adding the short-term interest rate as an additional explanatory variable into the mean equations. We included the lagged short-term interest rates, and the relevant coefficients (available upon request) are statistically insignificant at the 5% level in both stock and bond equations for each country, and thus the basic results remain similar.

We follow Patton (2006a) to search for the best copula to fit the data in our sample. We mainly compare the performance of static copulas versus the time-varying copula. We consider various shapes of copulas, including static copulas with different tail dependence structures and time-varying copulas, which allow the copula parameters to vary over time, thus providing more flexibility to the model building of the copula. Static copulas include Normal, Clayton, Frank, Gumbel, t , and SJC copulas; time-varying copulas include time-varying Normal and time-varying SJC copulas. Based on the information criteria (AIC and BIC) and the log-likelihood, the results (available on request) show that time-varying copulas outperform all static copulas. Nevertheless, the performance of the two time-varying copulas is close in terms of model fitting.

To better assess the adequacy of the two time-varying copula models fitted to the data and particularly those in extreme regions, we employ the hit test suggested by Patton (2006a) and divide the unit square into eight regions, as shown in Figure 1. In Table 2, we document the χ^2 statistic and the contribution of each region to the total value for each of the three countries. The time-varying Normal and SJC copula models pass the joint tests and all individual region tests at the 5% level. If, instead of the total χ^2 value, we examine the contribution of each particular region, the deficiencies of each copula would be clearly unveiled. To serve the purpose, we divide the unit square into eight regions, and examine a χ^2 statistic associated with each partition of the unit square. Since the sum of the proportions of the eight zones must be equal to one, we can match this χ^2 value against the 95% critical value for

seven degrees of freedom, which is 14.07. In particular, region one and two correspond to the lower-lower and upper-upper joint 10% tails for each variable. Region five is the median region. Panel (a) and (b) reveal that overall, the time-varying Normal copula is slightly better than the time-varying SJC copula in the U.S. and the UK because of their smaller χ^2 statistics. However, if we examine the contributions to the χ^2 statistics in regions one, two, and five, regions one and two show higher contributions in the χ^2 statistics for the Normal copula than the SJC copula, while region five shows lower contribution in the case of the Normal copula. This is expected since the Normal copula assumes zero tail dependence in both tails while the SJC copula allows potential upper-upper and lower-lower dependence. Obviously, the Normal copula fits better in the median region rather than in the extreme tail regions, as demonstrated from the lower contribution of χ^2 statistic in region five. Thus, if we focus on extreme tail regions one and two, the time-varying SJC copula also better fits the U.S. and the UK data. Panel (c) shows that overall, the time-varying SJC copula provides a better fit than the Normal copula in Germany because of the smaller χ^2 statistics. Furthermore, compared with the U.S. and the UK, the contributions of the χ^2 statistics in region one and two are even smaller in Germany, although the Normal copula still provides a better fit in region five in the case of Germany. In sum, the time-varying SJC copula provides a better fit for all three countries in regions one and two with extreme observations, which is of primary interest in this study.

3.4. Tail dependence coefficients

The presence of extreme value dependence can be informally assessed by transforming the original return series into standardized residuals so that they are on a common scale. The scatter plots of the standardized residuals for the stock and bond futures returns show that the U.S. and the UK exhibit mild tail dependence in the left-lower and right-upper corners, while most of the extreme points cluster at the right-lower and left-upper corners in the case of Germany. That is, stock and bond futures returns move in the opposite direction during extreme movements. The commonly used upper-upper and lower-lower tail dependence coefficients, which measure the movement of two random variables in the same direction at the extreme events, may not be adequate in the case of Germany. Instead, the lower-upper and upper-lower tail dependence coefficients will be considered here.

Figure 2 presents the time-varying lower-lower and upper-upper tail dependence coefficients based on the SJC copula for the U.S. and the UK. There are significant variations in the extreme correlations of stock and bond returns over the sample period. Two observations can be made here. First, the lower-lower tail dependence coefficients are generally higher than the upper-upper tail dependence coefficients, confirming the fact that financial markets seem more interdependent during bad times than during good times. Second, the magnitudes of the tail dependence coefficients decrease over the sample period, which also is consistent with the main finding of Yang, Zhou, and Wang (2009) that lower stock-bond (median) correlations tend to follow lower short-term interest rates. It is well known that short-term interest rates in the 2000s are much lower than those in the 1980s in both the U.S. and the UK.

Furthermore, the lower-upper and upper-lower tail dependence coefficients for both countries are equal to zero since no data point can be found in these regions.

Figure 3 shows the estimated lower-upper and upper-lower tail dependence coefficients for Germany. The patterns are very different compared with the U.S. and the UK. Again, the lower-upper tail dependence coefficient is higher than the upper-lower tail dependence coefficients. In other words, it is more likely for the bond market to inversely react to the stock market's downward movement than to the stock market's upward movement. The magnitudes of the tail dependence coefficients do not vary a lot over time.

3.5. Economic Significance of Asymmetric Correlations

The J statistic discussed in section 3.2 can only demonstrate the evidence of correlation asymmetry. A related question is how important these asymmetries may be from a viewpoint of the investor's portfolio decision-making, since statistically significant asymmetric correlation might not necessarily be economically important (and vice versa). Patton (2004) and Hong, Tu, and Zhou (2007) find that incorporating the information of asymmetric dependence leads to investors' gains that are economically significant. Okimoto (2008) also demonstrates that ignoring further symmetry in bear markets leads to substantial costs in risk management. On the other hand, Ang and Bekaert (2002) argue that the economic loss of not taking into account the difference between regimes of high and low dependence is small. In this section, following Patton (2004), we analyze the effect of asymmetric dependence between the stock and bond futures returns on the portfolio decision using the copula framework.

We assume that an investor only invests in two risky assets and invests a certain percentage of wealth $w_{x,t}$ in stock futures and $w_{y,t}$ in bond futures at time t , while the remainder of his wealth is assumed to be constant and held in cash for the ease of presentation.

By Sklar's theorem, the joint distribution between returns of two assets can be decomposed as $H_t(x,y) = C_t(F_t(x),G_t(y))$. A subscript t is added since the marginal distributions and the parameter in the copula function are time varying in this case.

Now, assume the investor has a *CRRA* utility function $U(W) = \frac{W^{1-\gamma}}{1-\gamma}$, where γ

measures the degree of relative risk aversion and W is the next period wealth.

$W_{t+1} = 1 + w_x X_{t+1} + w_y Y_{t+1}$ and X and Y denote the stock and bond future returns, respectively. The initial wealth is assumed to be $W_0=1$ and $(w_x, w_y) \in [0, 1]^2$ and $w_x + w_y \leq 1$. The investor problem is to determine the optimal weights $w_t^* \equiv [w_{x,t}^*, w_{y,t}^*]$ for his portfolio which maximize his expected utility function at

time t :

$$\begin{aligned} w_{t+1}^* &= \arg \max E_{\hat{H}_{t+1}} [U[1 + w_x X_{t+1} + w_y Y_{t+1}]] \\ &= \arg \max \iint U(1 + w_x x + w_y y) \cdot \hat{f}_{t+1}(x) \cdot \hat{g}_{t+1}(y) \cdot \hat{c}_{t+1}(\hat{F}_{t+1}(x), \hat{G}_{t+1}(y)) \cdot dx \cdot dy \end{aligned}$$

Since the parameters in the copula function are time varying, we employ a similar approach used by Patton (2004) to estimate the value of this double integral by 10,000 Monte Carlo replications to obtain the optimal values of w^* at each point of time. Further descriptions on the solving procedure can be found in the appendix in Patton (2004).

Following Ang and Chen (2002), we ask the utility loss to the investor who switches from asymmetric into symmetric copula, which represents the monetary compensation required for an investor to use the non-optimal normal weights w instead of the optimal asymmetric weights \tilde{w} . This estimate is given by $m = 100 \times (\bar{m} - 1)$, where $\bar{m} = (\tilde{U}/U)^{1/(1-\gamma)}$. \tilde{U} and \tilde{w} are the indirect CRRA utility and optimal weights under the asymmetric dependence model while U and w are the indirect CRRA utility and the sub-optimal weights under the asymmetric dependence model. $\tilde{U} = E[(\tilde{W})^{1-\gamma}]$ and $U = E[(W)^{1-\gamma}]$ for which $\tilde{W} = 1 + \tilde{w}_x x + \tilde{w}_y y$, $W = 1 + w_x x + w_y y$ and both the indirect CRRA functions are evaluated under the asymmetric dependence model.

Table 4 shows the effect of ignoring the asymmetric correlation. We assume that a constant relative risk aversion (CRRA) investor with risk aversion $\gamma = 1, 3, 7, 10, 20$ allocates his portfolio among stock futures, bond futures, and a riskless asset. The true distribution is based on the time-varying asymmetric SJC copula. The benchmark is the equally weighted scheme (the mix of 50:50) between stock and bond futures. Another candidate, which will be considered for comparison, is the time-varying normal copula. The table shows the utility losses or the advanced monetary compensation the investor should receive, in cents per dollar of wealth, if the investor holds sub-optimal normal weights or equal weights instead of the optimal asymmetric weights.

The results are somewhat mixed. For the U.S., the utility loss due to the sub-optimal normal weights increases from 2.25 cents to 8.76 cents as the risk

aversion parameter increase from 1 to 20, while in the case of equal weights, the asymmetry seems to have even stronger impact on the utility loss, ranging from 3.54 cents to 13.87 cents. The finding may be due to loss aversion becoming more important as the investor becomes more risk-averse. As a result, the utility loss, which ignores asymmetry, is greater. Adopting the equal weights scheme also provides less flexibility in choosing the optimal weights for the utility maximization, and thus leads to greater loss of utility. On the other hand, the utility loss due to the asymmetry in the UK and Germany is much smaller when compared to the U.S., with values less than three cents for the UK and one cent for Germany. In addition, the monotonic relation with the risk aversion parameter cannot be observed in these two markets. In fact, asymmetric dependence seems to have the smallest impact on Germany, which can be told from the small values of utility loss due to different values of risk aversion parameters. In summary, the economic cost of ignoring asymmetries is more significant in the U.S. than in the UK or Germany.

3.6. Regression Analysis

Thus far, we have an overall picture of how the stock-bond futures returns move at the extremes. In this subsection, we further investigate what drives their movement. Specifically, we explore whether the extreme correlation of stock-bond futures returns is related to macroeconomic news, stock market uncertainty and the business cycle. To measure stock market uncertainty, we use VIX and DVAX as proxies for the stock market uncertainties in the U.S. and two European countries, respectively. Business cycle data are obtained from the National Bureau of Economic Research (NBER) for

the U.S. and the Economic Cycle Research Institute (ECRI) for the UK and Germany. The indicator variable equals one (and zero otherwise) if the economy is in a recession, as defined by NBER and ECRI. Finally, we also explore the U.S. macroeconomic news announcements, which include news on nonfarm payroll, Producer Price Index (PPI), and the federal funds rate⁸. The news on nonfarm payroll and PPI is measured on the difference between the announced actual data and the consensus expectation about macroeconomic variables before the announcements, as reflected by the survey of dozens of economists and money managers, who provide their forecasts for a broad range of macroeconomic variables (e.g., Gunderson and Sterbenz, 2010). The news on the federal funds rate is measured by the difference between the announced federal funds target rate and the expectation based on the federal funds rate futures before the announcement (e.g., Wang, Yang, and Simpson, 2008). Table 3 shows more information about the U.S. macroeconomic news announcements.

Specifically, we regress the four tail dependence coefficients obtained earlier on all the variables mentioned before. We also compare the results with our median correlation regression.⁹ We perform several regressions as follows. First, we regress the four tails dependence coefficients (*TDC*) (upper-upper and lower-lower tail dependence coefficients are used for the U.S. and the UK while lower-upper and upper-lower tail dependence coefficients as the dependent variables for Germany) on

⁸ We also consider the effects of macroeconomic news surprise variables, but the regression results (see below) are not significant for both regression (a) and (b) when we use median correlation and tail dependence coefficients as the dependent variables. These results are not reported here.

⁹ We use the Normal copula to capture the median correlation since it assumes zero tail dependence in the model.

all the macroeconomic announcements' dummy variables:

$$(a) \quad TDC = a + \sum_{k=1}^3 b_k D_{kt} + e_t$$

Second, we investigate the effect of business cycles by including the recession dummy variable R_t both in the intercept and the slopes and run the following regression.

$$(b) \quad TDC = a + \sum_{k=1}^3 b_k D_{kt} + a^* R_t + \sum_{k=1}^3 b_k^* R_t D_{kt} + e_t$$

Column (a) of Table 5 reports the estimated parameters b_k for median correlation and the extreme correlations. All t -statistics are heteroskedasticity-adjusted based on White's robust standard errors, as OLS estimation residuals for some regression models below exhibit the heteroskedasticity problem. Most U.S. macroeconomic news variables are statistically significant in explaining the median correlation between stock-bond futures returns. Not surprisingly, the impacts are strongest in the case of the U.S., followed by the UK and Germany. It also is documented that U.S. macroeconomic announcements have a statistically significant impact on median stock-bond correlation in the UK and Germany. By contrast, we find only weak evidence for the relation between extreme correlations and U.S. macroeconomic announcements. To the best of our knowledge, such a sharp contrast has not yet been reported in the research.

Regarding the effect of the business cycle, we observe a statistically significantly different response of the stock-bond median correlation to the U.S. nonfarm payroll across the three countries during a recession in each country. Again, there is no differential effect related to the business cycle in the case of stock-bond extreme

correlations.

Finally, to understand the relation between stock-bond return extreme co-movement and stock market uncertainty, we follow existing studies (e.g., Kim, Moshirian, and Wu, 2006) and perform a regression on the stock-bond futures return extreme correlation on the stock market volatility. Connolly, Stivers, and Sun (2007) find that the change of implied volatility also affects the stock-bond co-movement. We add an interaction term in model (c) to assess the effect of business cycles on the TDC, and perform the following three regressions:

$$(c) \quad TDC_t = \beta_0 + \beta_1 \Delta \ln(uncert_t) + \beta_2 TDC_{t-1} + \beta_3 TDC_{t-2} + \mu_t$$

$$(d) \quad TDC_t = \beta_0 + \beta_1 \ln(uncert_{t-1}) + \beta_2 TDC_{t-1} + \beta_3 TDC_{t-2} + \mu_t$$

(e)

$$TDC_t = \beta_0 + \beta_1 \ln(uncert_{t-1}) + \beta_2 R_t + \beta_3 R_t * \ln(uncert_{t-1}) + \beta_4 TDC_{t-1} + \beta_5 TDC_{t-2} + \mu_t$$

TDC_t is the tail dependence coefficient measured at time t (U.S. and UK for the upper-upper and lower-lower tails while Germany for the lower-upper and upper-lower tails), $uncert_t$ indicates stock market uncertainty, which is measured by VIX for the U.S. and VDAX¹⁰ for the other two countries. R_t is the recession dummy. The purpose of the lag dependent variables is to capture the serial correlations in the error term. We also compare our results to the median correlation.

Column (a), (b), and (c) in Table 6 show the results of our model estimations.

Results from column (a) indicate that the coefficients of $\Delta \ln(uncert_t)$ have a

¹⁰ VIX is a popular measure of the implied volatility of Standard & Poor's 500-stock index options, while the VDAX indicates the implied volatility of the popular German stock market index DAX anticipated on the derivatives market.

negative relation with the *TDC* but are not statistically significant in most of the cases. The picture is different if we use $\ln(\text{uncert}_{t-1})$ as an explanatory variable in column (b). Typically, the estimated coefficient of $\ln(\text{uncert}_{t-1})$ is negative and statistically significant, which is consistent with the research and reflects the fact that uncertainty about the stock market leads to the “flight to quality” phenomenon. Column (c) demonstrates that the interaction terms related to recession are statistically significant in the case of the median correlation and the lower-lower (or lower-upper for Germany) tail correlation related to the stock market crash. If we compare the results across different dependent variables (median correlation, upper-upper, and lower-lower tails dependence), the results in column (b) reveal the stronger effects of stock market uncertainty on the stock-bond co-movement during extreme events than during the normal market period. For example, in the U.S., the coefficient of VIX is -0.003 for the stock-bond median correlation, but it decreases to -0.008 in the case of upper-upper tails dependence, and further decreases to -0.01 in the case of lower-lower tails dependence.

4. Conclusions

Using stock and bond futures data for three major developed markets (the U.S., the UK, and Germany) over last two decades, our study explores time-varying stock-bond futures’ extreme correlation using the copula method. There is evidence of positive extreme correlation between stock and bond futures returns in the U.S. and the UK when both markets are extremely bearish (lower-lower tail dependence) and

to a lesser extent extremely bullish (upper-upper tail dependence). The lower-lower (upper-upper) tail dependence between stock and bonds also is consistent with the observation that when inflation rates rise (decrease) both stock and bond markets might perform poorly (well), which is the essence of the Fed model (Siegel, 2002, p.104-107). Consistent with Yang, Zhou, and Wang (2009) regarding stock-bond (median) correlation in the U.S. and the UK, the extreme correlation also tends to be lower since the early 2000s, when short-term interest rates and inflation rates were low.

By contrast, German stock-bond futures extreme correlation is negative (upper-lower tail dependence and lower-upper dependence), which suggests enhanced diversification potential of one market to the other when either stock or bond futures market plunges in Germany. To the best of our knowledge, diagonal (i.e., lower-upper or upper-lower) tail dependence has not yet been applied in financial research, although it better measures the diversification potential. Also, lack of such diagonal tail dependence between stock and bond markets in the U.S. and the UK suggests somewhat limited diversification potential of bonds to stock investors in these countries during extreme events, which is presumably more likely to occur given longer investment horizon.

Furthermore, consistent with previous studies (e.g., Brenner, Pasquariello, and Subrahmanyam, 2009; Connolly, Stivers, and Sun, 2005; Andersen, Bollerslev, Diebold, and Vega, 2007), macroeconomic news announcements, the business cycle, and stock market uncertainty significantly affect the median stock-bond futures

correlation. However, only stock market uncertainty seems to significantly affect the extreme stock-bond futures correlation when the stock market is bearish, which has not yet been reported in the research. The finding also might be consistent with the documented importance of liquidity factors rather than macroeconomic fundamentals for lower quarterly returns in Baele, Bekaert, and Inghelbrecht (2010), as intuitively both stock market uncertainty and liquidity could become more important driving forces during a bearish stock market period.

Finally, it would be interesting to examine time-varying extreme correlations among stocks, bonds, and other asset classes both in a single country and a multi-country context, particularly allowing for both lower-upper and upper-lower tail dependences, which rarely have been addressed in the research. It also would be fruitful to explore how extreme correlations may be driven by domestic and international macroeconomic conditions, which may yield new insights different from existing studies focusing on median correlations.

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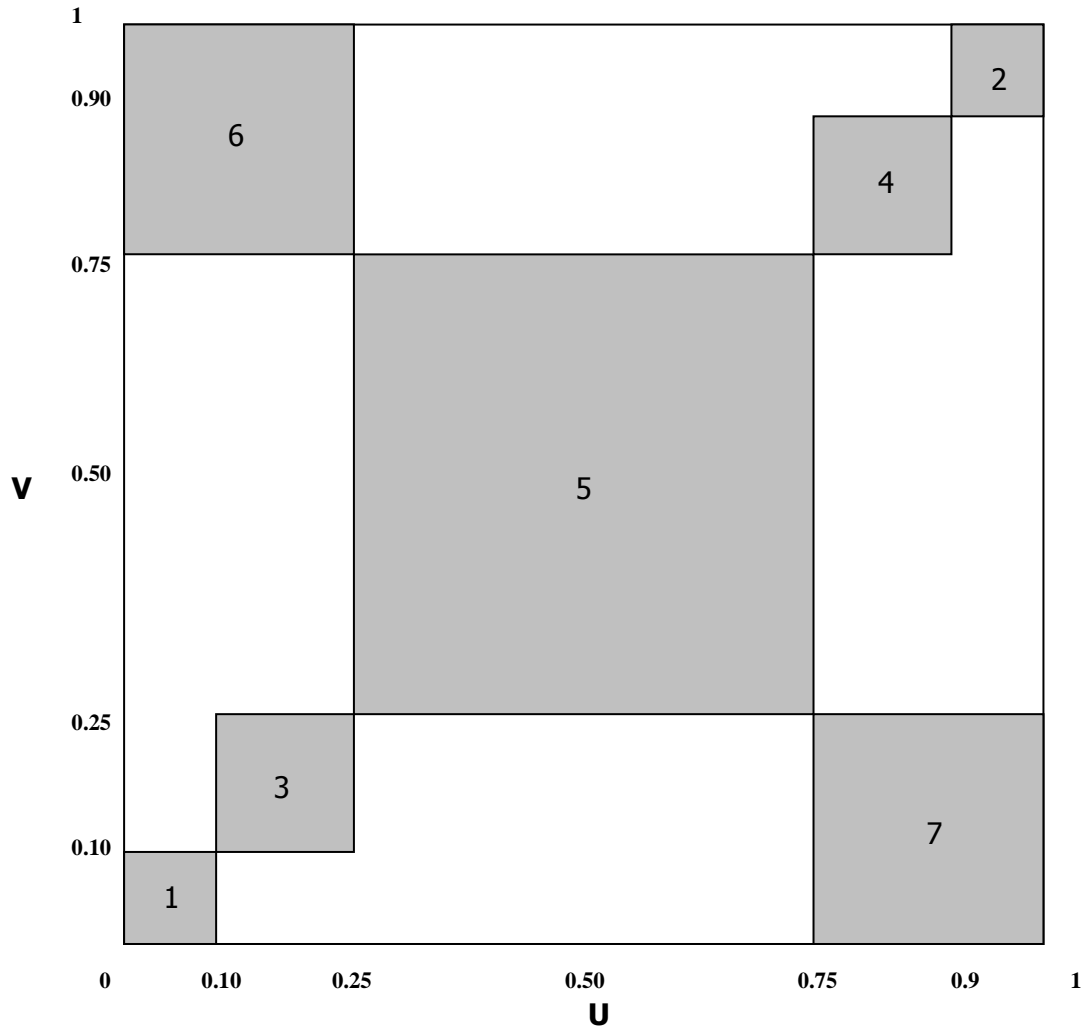


Figure 1. Regions for the hit test:

Region **1** and **2** correspond to the lower and upper joint 10% tails for each variable. Region **5** is the “median” region. Based on a χ^2 statistic associated to a partition of the unit square into eight zones. Since the sum of the proportions of the eight zones must equal to 1, we can match this χ^2 value against the 95% critical value for 7 degrees of freedom, which is 14.07.

US

UK

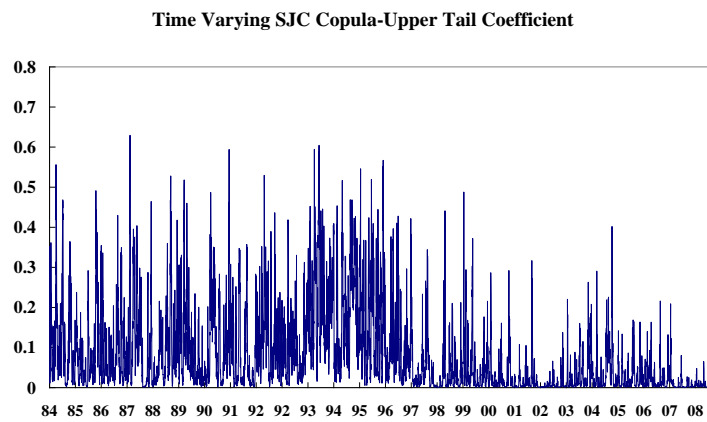
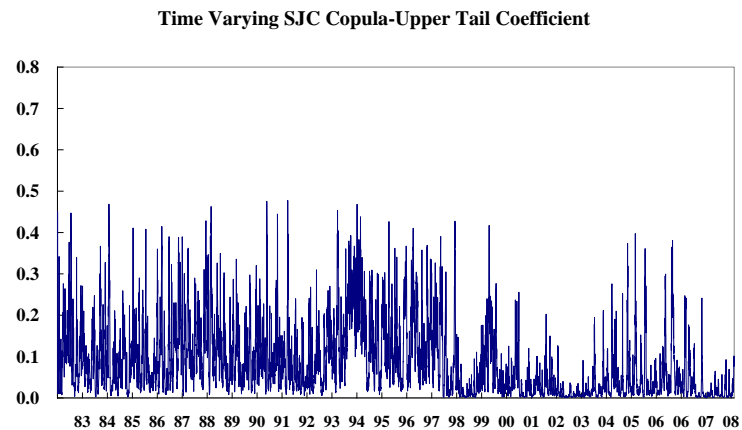
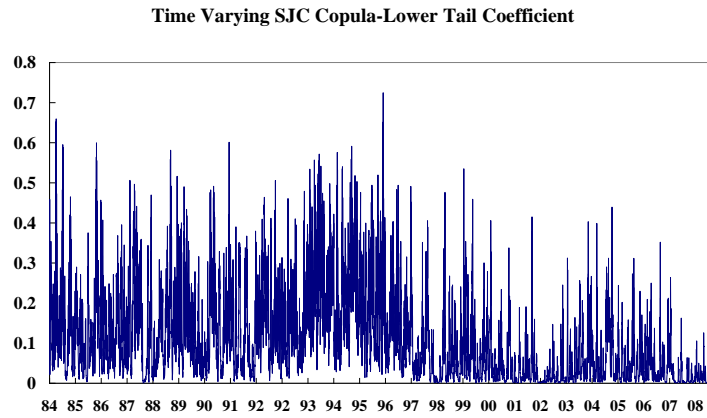
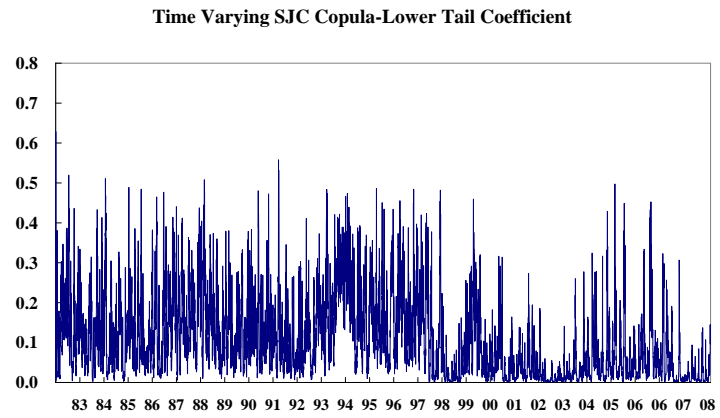
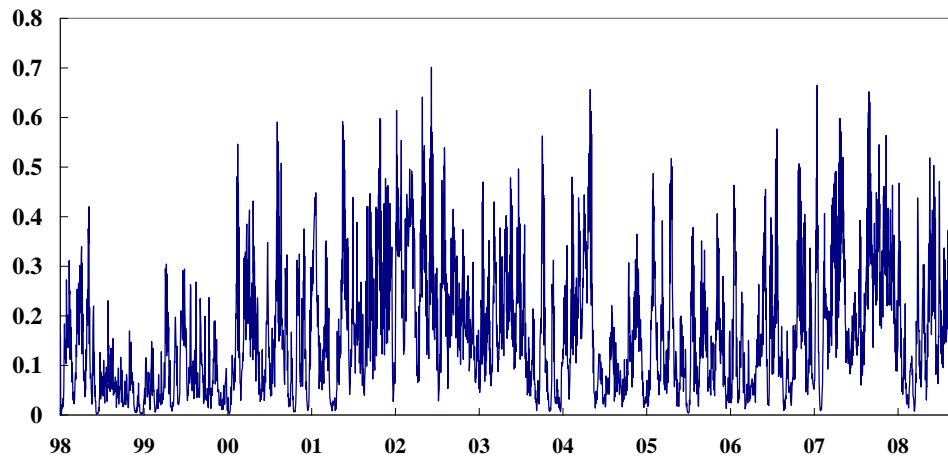


Figure 2. Lower-Lower and Upper-Upper tail dependence coefficients for the U.S. and the UK

**Time Varying SJC Copula-Lower(stock) Upper(bond) Tail
Coefficient**



**Time Varying SJC Copula-Upper(stock) Lower (bond) Tail
Coefficient**

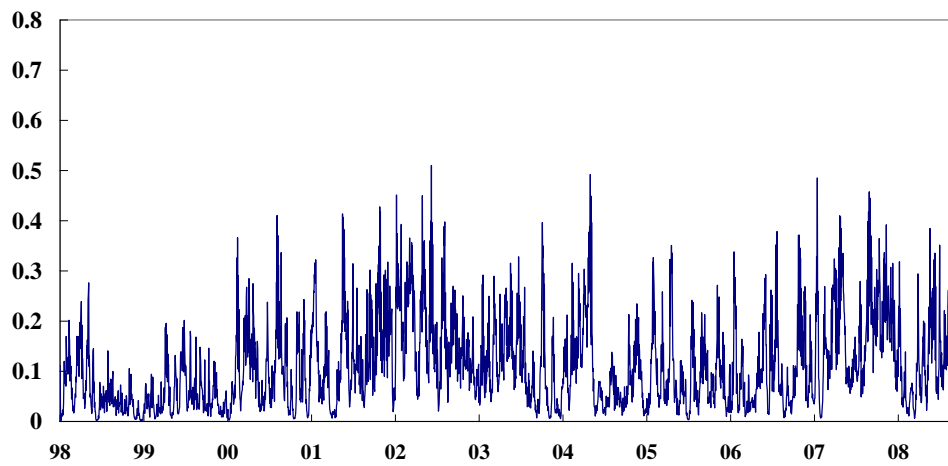


Figure 3. Lower-upper and upper-lower tail dependence coefficients for Germany

Table 1

Descriptive Statistics of stock index and bond futures returns for the U.S., the UK and Germany.

	US		UK		Germany	
	Stock	Bond	Stock	Bond	Stock	Bond
Mean	0.0001	0.0000	0.0001	0.0000	0.0000	0.0000
Std. Dev.	0.0054	0.0028	0.0051	0.0021	0.0072	0.0010
Skewness	-2.7562	-0.1537	-0.6601	0.2484	-0.1024	-0.4448
Kurtosis	99.4495	5.0335	15.4985	8.4203	9.8187	5.3957
Minimum	-0.1464	-0.0132	-0.0443	-0.0117	-0.0644	-0.0053
25% quantile	-0.0020	-0.0014	-0.0025	-0.0010	-0.0034	-0.0005
Median	0.0001	0.0000	0.0001	-0.0000	0.0001	0
75% quantile	0.0024	0.0016	0.0027	0.0011	0.0035	0.0006
Maximum	0.0771	0.0164	0.0614	0.0183	0.0524	0.0041
Jarque-Bera	2637321.0000	1195.2330	42344.9300	7942.4110	5181.0930	727.0634
<i>p</i> -value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
linear correlation	0.0923		0.1179		-0.3503	
Kendall Tau	0.1035		0.0920		-0.2109	

Table 2

Results of Hit test for stock index and bond futures returns in different countries.

(a)

Hit test for UK

Contribution to the χ^2 value (%) for different regions									
Model	χ^2	1	2	3	4	5	6	7	8
Time-varying Normal	5.32	25.65	18.86	5.34	5.29	5.63	4.16	4.31	31.20
Time-varying SJC	12.37	12.19	8.28	7.21	4.56	23.56	4.17	3.58	36.70

(b)

Hit test for U.S.

Contribution to the χ^2 value (%) for different regions									
Model	χ^2	1	2	3	4	5	6	7	8
Time-varying Normal	5.47	25.34	15.21	6.37	4.52	10.18	5.21	3.72	29.51
Time-varying SJC	7.68	10.21	4.28	5.27	3.26	25.41	5.72	4.16	41.70

(c)

Hit test for Germany (Note: the square is rotated 90 degrees clockwise)

Contribution to the χ^2 value (%) for different regions									
Model	χ^2	1	2	3	4	5	6	7	8
Time-varying Normal	12.17	12.76	14.61	4.32	7.85	11.25	6.78	7.43	34.80
Time-varying SJC	2.18	3.24	4.71	6.87	8.29	20.56	5.76	6.43	43.70

Table 3
U.S. Macroeconomic News Announcements

Announcement	Obs.	Source	Dates
Nonfarm Payroll Employment	348	BLS	1/1980-12/2008
Production Price Index	348	BLS	1/1980-12/2008
Target Federal fund rate	132	FRB	2/1993-12/2008

Note: BLS represents Bureau of Labor Statistics and FRB Federal Reserve Board

Table 4. The Economic Cost of Asymmetric Correlations.

Table 4 shows the effect of ignoring the asymmetric correlation. A constant relative risk aversion (CRRA) investor with risk aversion $\gamma = 1, 3, 7, 10, 20$ allocates his portfolio among the stock, bond futures, and a riskless asset. The true distribution is based on the time-varying asymmetric SJC copula. The benchmark is the equally weighted scheme (50:50 mix) and time-varying normal copula is symmetric in nature. The table shows the utility losses, in cents per dollar of wealth, created if the investor holds sub-optimal normal weights or equal weights instead of the optimal asymmetric weights.

U.S.					
γ	1	3	7	10	20
50:50 mix	3.5421	5.671	7.353	10.452	13.871
Normal copula	0.2521	1,923	2,871	5.621	8.762
U.K.					
γ	1	3	7	10	20
50:50 mix	3.182	2.174	3.752	4.519	4.712
Normal copula	1.254	0.962	2.178	2.741	2.072
Germany					
γ	1	3	7	10	20
50:50 mix	2.120	5.168	2.195	3.217	4.127
Normal copula	0.324	0.210	1.037	0.537	0.746

Table 5

Regression results of the impact of macroeconomic news announcements on stock-bond futures return correlation.

	US			UK			Germany		
	(a) b_k	(b) b_k b_k^*		(a) b_k	(b) b_k b_k^*		(a) b_k	(b) b_k b_k^*	
Median correlation									
Nonfarm Payroll	0.201 **	0.212 ***	-0.094 **	0.126 **	0.151 *	-0.053 **	0.075 *	0.082 *	-0.031 *
PPI	0.117	0.148	-0.049	0.107 *	0.135	-0.027	-0.115 *	0.122	-0.019
Fed Fund rate	-0.053 *	-0.056 *	0.017	0.020	0.019	-0.015	0.028	0.039	-0.010
Adj R ²	0.046	0.077		0.035	0.053		0.031	0.044	
Q test $\sim \chi_{20}^2$	22.048	14.976		36.510 *	23.091		14.102	28.741	
Upper-Upper tails									
Nonfarm Payroll	0.059	0.053	0	0.031	0.035	-0.006	-0.037	-0.035	0
PPI	-0.034	-0.035	0.001	0.036	0.043	-0.004 *	-0.028	-0.03	0.004
Fed Fund rate	0.011	0.012	0	0.019	0.022	-0.002	0.019	0.022	-0.003
Adj R ²	0.011	0.022		0.016	0.030		0.013	0.034	
Q test $\sim \chi_{20}^2$	20.189	37.871 **		21.652	29.152		37.210 *	19.219	
Lower-Lower tails									
Nonfarm Payroll	0.064	0.061	-0.002	0.039	0.042	-0.007	-0.040	-0.043	-0.003
PPI	-0.038	-0.040	0.003	0.041	0.047	-0.006	-0.031	-0.04	-0.004
Fed Fund rate	0.015	0.017	-0.005	0.025	0.027	-0.006	0.024	0.028	-0.004
Adj R ²	0.023	0.037		0.030	0.044		0.029	0.046	
Q test $\sim \chi_{20}^2$	21.170	20.011		28.002	21.103		26.412	29.193	

Notes: For Germany the table of lower-lower tails means lower (stock)-upper (bond) tails. The table of upper-upper tails means upper (stock)-lower (stock) tails. Ljung-Box Q test is for a null hypothesis of no serial correlation in the disturbance terms up to the 20th order. t -statistics are heteroskedasticity-adjusted based on White's robust standard errors. The symbol *** indicates statistical significance at the 0.001 level, ** at the 0.01 level, and * at the 0.05 level.

Table 6

Regression results of the impact of stock market uncertainties on stock-bond futures return correlation

	US			UK			Germany		
Median correlation	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
$\Delta VIX(\Delta VDAX)$	-0.007			-0.007			-0.012		
$VIX(VDAX)_{t-1}$		-0.003 ***	-0.005 ***		-0.003 ***	-0.005 ***		-0.005 *	-0.003
$VIX(VDAX)_{t-1} * \text{Recession}$			0.016 **			0.016 **			-0.003
Adj R ²	0.992	0.992	0.992	0.991	0.991	0.991	0.978	0.978	0.978
Q test $\sim \chi_{20}^2$	27.355	25.769	20.571	37.959 **	19.370	20.127	17.976	35.800 *	28.204
Upper-Upper tails									
$\Delta VIX(\Delta VDAX)$	-0.004			-0.004			0.012		
$VIX(VDAX)_{t-1}$		-0.008 ***	-0.008 ***		-0.008 ***	-0.008 ***		0.006 *	0.003
$VIX(VDAX)_{t-1} * \text{Recession}$			0.09			0.09			0.007
Adj R ²	0.836	0.836	0.837	0.774	0.774	0.775	0.693	0.694	0.694
Q test $\sim \chi_{20}^2$	31.287	31.879 *	19.362	19.362	28.622	32.913 *	30.385	21.317	14.016
Lower-Lower tails									
$\Delta VIX(\Delta VDAX)$	-0.026 *			-0.026 *			0.002		
$VIX(VDAX)_{t-1}$		-0.01 **	-0.012 ***		-0.01 **	-0.012 ***		0.009 *	0.004
$VIX(VDAX)_{t-1} * \text{Recession}$			0.024 *			0.024 *			0.01
Adj R ²	0.761	0.761	0.762	0.725	0.725	0.726	0.685	0.685	0.686
Q test $\sim \chi_{20}^2$	20.620	21.148	29.871	26.257	18.154	23.214	21.742	31.528 *	26.124

Notes: For the U.S., we use VIX; for UK and Germany, we use DVAX. Recession is a dummy variable=1 if recession, 0=expansion, ΔVIX is the first difference of VIX. For Germany the table of lower-lower tails means lower (stock)-upper (bond) tails. The table of upper-upper tails means upper (stock)-lower (stock) tails. Ljung-Box Q test is for a null hypothesis of no serial correlation in the disturbance terms up to the 20th order. t -statistics are heteroskedasticity-adjusted based on White's robust standard errors. The symbol *** indicates statistical significance at the 0.001 level, ** at the 0.01 level, and * at the 0.05 level.